Package: markovDP (via r-universe)

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Title Infrastructure for Discrete-Time Markov Decision Processes (MDP)

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Description Provides the infrastructure to work with Markov Decision Processes (MDPs) in R. The focus is on convenience in formulating MDPs, the support of sparse representations (using sparse matrices, lists and data.frames) and visualization of results. Some key components are implemented in C++ to speed up computation. Several popular solvers are implemented.

License GPL (>=3)

URL https://github.com/mhahsler/markovDP

BugReports https://github.com/mhahsler/markovDP/issues

Depends R (>= 3.5.0)

Imports fastmap, foreach, igraph, lpSolve, Matrix, MatrixExtra, methods, progress, Rcpp, stats

Suggests doParallel, gifski, knitr, rmarkdown, testthat, visNetwork

LinkingTo Rcpp

VignetteBuilder knitr

Classification/ACM G.4, G.1.6, I.2.6

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Collate 'AAA_check_installed.R' 'AAA_colors.R' 'AAA_foreach_helper.R' 'AAA_nodots.R' 'AAA_package.R' 'AAA_progress.R' 'AAA_sample_sparse.R' 'AAA_shorten.R' 'AAA_which_max_random.R' 'Cliff_walking.R' 'DynaMaze.R' 'MDP.R' 'MDPTF.R' 'Maze.R' 'Q_values.R' 'RcppExports.R' 'Windy_gridworld.R' 'absorbing_states.R' 'accessors_transitions.R'

'accessors_reward.R' 'accessors.R' 'act.R' 'action.R'
'action_state_helpers.R' 'available_actions.R'
'bellman_operator.R' 'check_and_fix_MDP.R'
'convergence_horizon.R' 'find_reachable_states.R' 'greedy.R'
'gridworld.R' 'policy.R' 'policy_evaluation.R'
'policy_evaluation_LP.R' 'regret.R' 'reward.R'
'round_stochchastic.R' 'sample_MDP.R' 'sample_MDPTF.R'
'solve_MDP.R' 'solve_MDP_APPROX.R' 'solve_MDP_DP.R'
'solve_MDP_LP.R' 'solve_MDP_MC.R' 'solve_MDP_SAMP.R'
'solve_MDP_TD.R' 'sparse_helpers.R' 'start.R'
'transition_graph.R' 'unreachable_states.R' 'value_function.R'
'visit_probability.R' 'zzz.R'

Config/pak/sysreqs libglpk-dev libxml2-dev

Repository https://mhahsler.r-universe.dev

RemoteUrl https://github.com/mhahsler/markovDP

RemoteRef HEAD

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Contents

`absorbing_states` *Absorbing States*

Description

Find absorbing states using the transition model.

Usage

```
absorbing_states(model, state = NULL, ...)

## S3 method for class 'MDP'
absorbing_states(
  model,
  state = NULL,
  sparse = "states",
  use_precomputed = TRUE,
  ...
)

## S3 method for class 'MDPTF'
absorbing_states(
  model,
  state = NULL,
  sparse = "features",
  use_precomputed = TRUE,
  ...
)
```

Arguments

model	a MDP object.
state	a single state to check. This can be much faster if the model contains a transition model implemented as a function. NULL means all states are checked.
...	further arguments are passed on.
sparse	logical; if return a sparse logical vector?
use_precomputed	logical; should precomputed values in the MDP be used?

Details

The function absorbing_states() checks if a state or a set of states are absorbing (terminal states). A state is absorbing if there is for all actions a probability of 1 for staying in the state.

Value

absorbing_states() returns a logical vector indicating if the states are absorbing (terminal).

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other MDPTF: MDPTF(), act(), sample_MDP.MDPTF(), solve_MDP(), solve_MDP_APPROX(), start()

Examples

```
data(Maze)

gw_matrix(Maze)
gw_matrix(Maze, what = "labels")
gw_matrix(Maze, what = "absorbing")

# -1 and +1 are absorbing states
absorbing_states(Maze)
absorbing_states(Maze, sparse = FALSE)
absorbing_states(Maze, sparse = "states")

# check individual states
absorbing_states(Maze, "s(1,1)")
absorbing_states(Maze, "s(1,4)")
```

act *Perform an Action*

Description

Performs an action in a state and returns the new state and reward.

Usage

```
act(model, state, action, fast = FALSE, ...)

## S3 method for class 'MDP'
act(model, state, action = NULL, fast = FALSE, ...)

## S3 method for class 'MDPTF'
act(model, state, action, fast = FALSE, ...)
```

Arguments

model	an MDP model.
state	the current state.
action	the chosen action. If the action is not specified (NULL) and the MDP model contains a policy, then the action is chosen according to the policy.
fast	logical; if TRUE then extra state id to label conversions are avoided.
...	if action is unspecified, then the additional parameters are passed on to action() to determine the action using the model's policy.

Value

a names list with the reward and the next state_prime.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other MDPTF: MDPTF(), absorbing_states(), sample_MDP.MDPTF(), solve_MDP(), solve_MDP_APPROX(), start()

Examples

```
data(Maze)

act(Maze, "s(1,3)", "right")

# solve the maze and then ask for actions using the policy
sol <- solve_MDP(Maze)
act(sol, "s(1,3)")

# make the policy in sol epsilon-soft and ask 10 times for the action
replicate(10, act(sol, "s(1,3)", epsilon = .2))
```

---

action *Choose an Action Given a Policy*

Description

Returns an action given a deterministic policy. The policy can be made epsilon-soft.

Usage

```
action(model, state, epsilon = 0, epoch = 1, ...)
```

Arguments

model	a solved MDP.
state	the state.
epsilon	make the policy epsilon soft.
epoch	what epoch of the policy should be used. Use 1 for converged policies.
...	further parameters are passed on.

Value

The name of the optimal action as a factor.

Author(s)

Michael Hahsler

See Also

Other policy: Q_values(), bellman_update(), greedy_action(), policy(), policy_evaluation(), regret(), reward(), value_function(), visit_probability()

Examples

```
data("Maze")
Maze

sol <- solve_MDP(Maze)
policy(sol)

action(sol, state = "s(1,3)")

## choose from an epsilon-soft policy
table(replicate(100, action(sol, state = "s(1,3)", epsilon = 0.1)))
```

---

action_state_helpers *Conversions for Action and State IDs and Labels*

Description

Several helper functions to convert state and action (integer) IDs to labels and vice versa.

Usage

```
normalize_state(state, model)

normalize_state_id(state, model)

normalize_state_label(state, model)

normalize_action(action, model)

normalize_action_id(action, model)

normalize_action_label(action, model)

state2features(state)

features2state(x)

s(...)

normalize_state_features(state, model = NULL)
```

Arguments

state	a state labels
model	a MDP model
action	a action labels
x	a state feature vector or a matrix of state feature vectors as rows.
...	features that should be converted into a row vector used to describe a state.

Details

`normalize_state()` and `normalize_action()` convert labels or ids into a factor which is the standard representation. If only the label or the integer id (i.e., the index) is needed, the additional functions can be used. These are typically a lot faster.

To support a factored state representation as feature vectors, `state2features()` and `feature2states()` are provided. A factored state is represented as a **row** vector for a single state (ceonveniently created via `s()`) or a matrix with row vectors for a set of states are used. State labels are constructed in the form `s(feature1, feature2, ...)`. Factored state representation is used for value function approximation (see `solve_MDP_APPROX()`) and for MDPTF to describe MDP's via a transition function between factored states.

Value

Functions ending in

- `_id` return an integer id,
- `_label` return a character string,
- `_features` return a state feature matrix,
- no ending return a factor.

Other functions:

- `state2features()` returns a feature vector/matrix.
- `features2state(x)` returns a state label in the format `s(feature list)`.
- `s()` returns a state features row vector.

`available_actions` *Available Actions in a State*

Description

Determine the set of actions available in a state.

Usage

```
available_actions(model, state, neg_inf_reward = TRUE, stay_in_place = FALSE)
```

Arguments

`model`	a MDP object.
`state`	a character vector specifying the states.
`neg_inf_reward`	logical; consider an action that produced `-Inf` reward to all end states unavailable?
`stay_in_place`	logical; consider an action that results in the same state with a probability of 1 as unavailable. Note that this means that absorbing states have no available action!

Details

Unavailable actions are modeled as actions that have an immediate reward of `-Inf` in the reward function. For a maze, also actions that do not change the state can be considered unavailable.

Value

a character vector with the available actions.

a vector with the available actions.

Author(s)

Michael Hahsler

See Also

Other MDP: `MDP()`, `Q_values()`, `absorbing_states()`, `act()`, `bellman_update()`, `convergence_horizon()`, `greedy_action()`, `gridworld`, `policy_evaluation()`, `regret()`, `sample_MDP()`, `solve_MDP()`, `start()`, `transition_graph()`, `transition_matrix()`, `unreachable_states()`, `value_function()`

Examples

```
data(DynaMaze)
gw_plot(DynaMaze)

# The the following actions are always available:
DynaMaze$actions

# only right and down is unavailable for s(1,1) because they
#   make the agent stay in place.
available_actions(DynaMaze, state = "s(1,1)", stay_in_place = TRUE)

# An action that leaves the grid currently is allowed but does not do
# anything.
act(DynaMaze, "s(1,1)", "up")
```

---

bellman_update *Bellman Update and Bellman operator*

Description

Update the value function with a Bellman update.

Usage

```
bellman_update(model, V)

bellman_operator(model, pi, V)
```

Arguments

model	an MDP problem specification.
V	a vector representing the value function. A single 0 can be used as a shorthand for a value function with all 0s.
pi	a policy as a data.frame with at least columns for states and action. If NULL, then the policy in model is used.

Details

The Bellman update updates a value function given the model by applying the Bellman equation as an update rule for each state:

$$v_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}(s)} \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v_k(s')]$$

The Bellman update moves the estimated value function V closer to the optimal value function v_*.

The Bellman operator B_π updates a value function given the model, and a policy π:

$$(B_\pi v)(s) = \sum_{a \in \mathcal{A}} \pi(a|s) \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v(s')]$$

The Bellman error is $\delta = B_\pi v - v$. The Bellman operator reduces the Bellman error and moves the value function closer to the fixed point of the true value function:

$$v_\pi = B_\pi v_\pi.$$

Value

a list with the updated state value vector U and the taken actions pi.

Author(s)

Michael Hahsler

References

Sutton, R. S., Barto, A. G. (2020). Reinforcement Learning: An Introduction. Second edition. The MIT Press.

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other policy: Q_values(), action(), greedy_action(), policy(), policy_evaluation(), regret(), reward(), value_function(), visit_probability()

Examples

```
data(Maze)
Maze

# single Bellman update from a all 0 value function
bellman_update(Maze, V = 0)

# perform simple value iteration for 10 iterations
V <- 0
for (i in seq(10))
  V <- bellman_update(Maze, V)$V

V
```

`Cliff_walking` *Cliff Walking Gridworld MDP*

Description

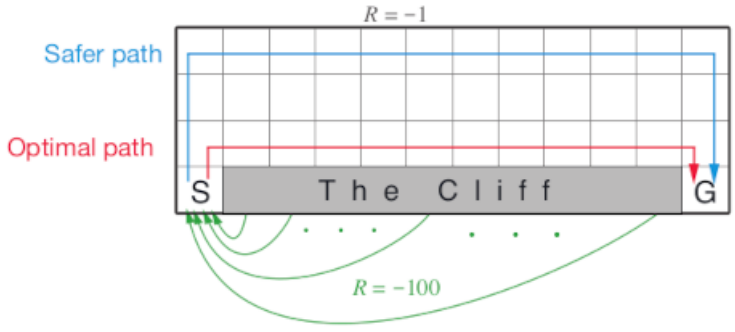
The cliff walking gridworld MDP example from Chapter 6 of the textbook "Reinforcement Learning: An Introduction."

Format

An object of class MDP.

Details

The cliff walking gridworld has the following layout:



The gridworld is represented as a 4 x 12 matrix of states. The states are labeled with their x and y coordinates. The start state is in the bottom left corner. Each action has a reward of -1, falling off the cliff has a reward of -100 and returns the agent back to the start. The episode is finished once the agent reaches the absorbing goal state in the bottom right corner. No discounting is used (i.e., $\gamma = 1$).

References

Richard S. Sutton and Andrew G. Barto (2018). Reinforcement Learning: An Introduction Second Edition, MIT Press, Cambridge, MA.

See Also

Other MDP_examples: DynaMaze, MDP(), Maze, Windy_gridworld

Other gridworld: DynaMaze, Maze, Windy_gridworld, gridworld

Examples

```
data(Cliff_walking)
Cliff_walking

gw_matrix(Cliff_walking)
gw_matrix(Cliff_walking, what = "labels")

# The Goal is an absorbing state
absorbing_states(Cliff_walking, sparse = "states")

# visualize the transition graph
gw_plot_transition_graph(Cliff_walking)

# solve using different methods
sol <- solve_MDP(Cliff_walking)
sol
policy(sol)
gw_plot(sol)
```

---

colors *Default Colors for Visualization*

Description

Default discrete and continuous colors used in the package markovDP.

Usage

```
colors_discrete(n, col = NULL)

colors_continuous(val, col = NULL)
```

Arguments

n	number of states.
col	custom color palette. colors_discrete() uses the first n colors. colors_continuous() uses the given colors to calculate a palette (see grDevices::colorRamp()). The default is a blue-red color ramp.
val	a vector with values to be translated to colors.

Value

`colors_discrete()` returns a color palette and `colors_continuous()` returns the colors associated with the supplied values.

Examples

```
colors_discrete(5)

colors_continuous(runif(10))
```

`convergence_horizon` *Estimate the Convergence Horizon for an Infinite-Horizon MDP*

Description

Many sampling-based methods require a finite horizon. Fo infinite horizons, discounting leads to convergences during a finite horizon. This function estimates the number of steps till convergence using rules of thumb.

Usage

```
convergence_horizon(model, delta = 0.001, n_updates = 10)
```

Arguments

`model`	an MDP model.
`delta`	maximum update error.
`n_updates`	integer; average number of time each state is updated.

Details

The horizon is estimated differently for the discounted and the undiscounted case.

Discounted Case:

The effect of the largest reward $R_{\max}$ update decreases with t as $\delta_t = \gamma^t R_{\max}$. The convergence horizon is estimated as the smallest t for which $\delta_t < \delta$.

Undiscounted Case:

For the undiscounted case, episodes end when an absorbing state is reached. It cannot be guaranteed that a model will reach an absorbing state. To avoid infinite loops, we set the maximum horizon such that each entry in the Q-table is on average updated `n_updates` times. This is a very rough rule ot thumb.

Value

An estimated convergence horizon.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Examples

```
data(Maze)
Maze

convergence_horizon(Maze)

# make the Maze into a discounted problem where future rewards count less.
Maze_discounted <- Maze
Maze_discounted$discount <- .9
Maze_discounted

convergence_horizon(Maze_discounted)
```

DynaMaze *The Dyna Maze*

Description

The Dyna Maze from Chapter 8 of the textbook "Reinforcement Learning: An Introduction."

Format

An object of class MDP.

Details

The simple 6x9 maze with a few walls.

References

Richard S. Sutton and Andrew G. Barto (2018). Reinforcement Learning: An Introduction Second Edition, MIT Press, Cambridge, MA.

See Also

Other MDP_examples: Cliff_walking, MDP(), Maze, Windy_gridworld

Other gridworld: Cliff_walking, Maze, Windy_gridworld, gridworld

Other MDP_examples: Cliff_walking, MDP(), Maze, Windy_gridworld

Other gridworld: Cliff_walking, Maze, Windy_gridworld, gridworld

Examples

```
data(DynaMaze)

DynaMaze

gw_matrix(DynaMaze)
gw_matrix(DynaMaze, what = "labels")

gw_plot_transition_graph(DynaMaze)
```

---

find_reachable_states *Find Reachable State Space from a Transition Model Function*

Description

Finds the reachable state space from a transition model function that takes the arguments `model`, `action`, `start.state` and returns a named vector with the probabilities for the resulting end.states (typically only the ones with a probability greater than 1).

Usage

```
find_reachable_states(
  transition_function,
  start_state,
  actions,
  model = NULL,
  horizon = Inf,
  progress = TRUE
)
```

Arguments

`transition_function`	a transition function (see details for requirements).
`start_state`	labels of the start states.
`actions`	a vector with the available actions.
`model`	if needed, the model passed on to the transition model function.
`horizon`	only return states reachable in the given horizon.
`progress`	logical; show a progress bar?

Details

The function performs a (depth-limited) depth-first traversal of the search state space and returns a vector with the names of all encountered states. This vector can be used as the states for creating a MDP model.

The transition function needs to be a function with the argument list `model`, `action`, `start.state` which returns named vector only containing the non-zero probabilities named by the corresponding end state. A partial model that contains `actions` and `start.state` will be supplied to the function.

Value

a character vector with all reachable states.

Author(s)

Michael Hahsler

Examples

```
# define a MDP for Tic-Tac-Toe

# state description: matrix with the characters _, x, and o
#                    can be converted into a label of 9 characters

# set of actions
A <- as.character(1:9)

# helper functions
ttt_empty_board <- function() matrix('_', ncol = 3, nrow = 3)

ttt_state2label <- function(state) paste(state, collapse = '')

ttt_label2state <- function(label) matrix(strsplit(label, "")[[1]],
                                          nrow = 3, ncol = 3)

ttt_available_actions <- function(state) {
  if (length(state) == 1L) state <- ttt_label2state(state)
  which(state == "_")
}

ttt_result <- function(state, player, action) {
  if (length(state) == 1L) state <- ttt_label2state(state)

  if (state[action] != "_")
    stop("Illegal action.")

  state[action] <- player
  state
}

ttt_terminal <- function(state) {
  if (length(state) == 1L) state <- ttt_label2state(state)

  # Check the board for a win and return one of
  # 'x', 'o', 'd' (draw), or 'n' (for next move)
  win_possibilities <- rbind(state,
                             t(state),
                             diag(state),
                             diag(t(state)))

  wins <- apply(win_possibilities, MARGIN = 1, FUN = function(x) {
    if (x[1] != '_' && length(unique(x)) == 1) x[1]
```

```
    else '_'
  })

  if (any(wins == 'x'))
    return('x')

  if (any(wins == 'o'))
    return('o')

  # Check for draw
  if (sum(state == '_') < 1)
    return('d')

  return('n')
}

# define the transition function:
#     * return a probability vector for an action in a start state
#     * we define the special states 'win', 'loss', and 'draw'
P <- function(model, action, start.state) {
  action <- as.integer(action)

  # absorbing states
  if (start.state %in% c('win', 'loss', 'draw', 'illegal')) {
    return(structure(1, names = start.state))
  }

  # avoid illegal action by going to the very expensive illegal state
  if (!(action %in% ttt_available_actions(start.state))) {
    return(structure(1, names = "illegal"))
  }

  # make x's move
  next_state <- ttt_result(start.state, 'x', action)

  # terminal?
  term <- ttt_terminal(next_state)
  if (term == 'x') {
    return(structure(1, names = "win"))
  } else if (term == 'o') {
    return(structure(1, names = "loss"))
  } else if (term == 'd') {
    return(structure(1, names = "draw"))
  }

  # it is o's turn
  actions_of_o <- ttt_available_actions(next_state)
  possible_end_states <- lapply(
    actions_of_o,
    FUN = function(a)
      ttt_result(next_state, 'o', a)
  )
```

```
  # fix terminal states
  term <- sapply(possible_end_states, ttt_terminal)
  possible_end_states <- sapply(possible_end_states, ttt_state2label)
  possible_end_states[term == 'x'] <- 'win'
  possible_end_states[term == 'o'] <- 'loss'
  possible_end_states[term == 'd'] <- 'draw'

  possible_end_states <- unique(possible_end_states)

  return(structure(rep(1 / length(possible_end_states),
                       length(possible_end_states)),
                   names = possible_end_states))
}

# define the reward
R <- rbind(
  R_(                    value = 0),
  R_(end.state = 'win',  value = +1),
  R_(end.state = 'loss', value = -1),
  R_(end.state = 'draw', value = +.5),
  R_(end.state = 'illegal', value = -Inf),
  # Note: there is no more reward once the agent is in a terminal state
  R_(start.state = 'win',  value = 0),
  R_(start.state = 'loss', value = 0),
  R_(start.state = 'draw', value = 0),
  R_(start.state = 'illegal', value = 0)
)

# start state
start <- ttt_state2label(ttt_empty_board())
start

# find the reachable state space
S <- union(c('win', 'loss', 'draw', 'illegal'),
          find_reachable_states(P, start_state = start, actions = A))
head(S)

tictactoe <- MDP(S, A, P, R, discount = 1, start = start, name = "TicTacToe")
tictactoe
```

`greedy_action` *Greedy Actions and Policies*

Description

Extract a greedy policy or select a greedy action from a solved model or a Q matrix.

Usage

```
greedy_action(x, s, Q = NULL, epsilon = 0, prob = FALSE)
```

```
greedy_policy(x)
```

Arguments

x	a solved MDP model or a Q matrix.
s	a state.
Q	an optional Q-matrix.
epsilon	an epsilon > 0 applies an epsilon-greedy policy.
prob	logical; return a probability distribution over the actions.

Value

- greedy_action() returns the action with the highest q-value for state s. If prob = TRUE, then a vector with the probability for each action is returned.
- greedy_policy() returns a data.frame with the policy.

greedy_policy() returns the greedy policy given Q.

Author(s)

Michael Hahsler

References

Sutton, R. S., Barto, A. G. (2020). Reinforcement Learning: An Introduction. Second edition. The MIT Press.

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other policy: Q_values(), action(), bellman_update(), policy(), policy_evaluation(), regret(), reward(), value_function(), visit_probability()

Examples

```
data(Maze)
Maze

# create a random policy and calculate q-values
pi_random <- random_policy(Maze)
pi_random

V <- policy_evaluation(Maze, pi_random)
V

# calculate Q values
Q <- Q_values(Maze, V)
```

```
Q

# get the greedy policy form the Q values
pi_greedy <- greedy_policy(Q)
pi_greedy
Maze_with_policy <- add_policy(Maze, pi_greedy)
gw_plot(Maze_with_policy, main = "Maze: Greedy Policy")

# find the greedy/ epsilon-greedy action for the top-left corner state
greedy_action(Maze, "s(1,1)", Q, epsilon = 0, prob = FALSE)
greedy_action(Maze, "s(1,1)", Q, epsilon = 0, prob = TRUE)
greedy_action(Maze, "s(1,1)", Q, epsilon = .1, prob = TRUE)

# we can also specify a model with a policy and use the internal Q-values
greedy_action(Maze_with_policy, "s(1,1)", epsilon = .1, prob = TRUE)
```

---

gridworld *Helper Functions for Gridworld MDPs*

Description

Helper functions for gridworld MDPs to convert between state names and gridworld positions, and for visualizing policies.

Usage

```
gw_init(
  dim,
  actions = c("up", "right", "down", "left"),
  start = NULL,
  goal = NULL,
  absorbing_states = NULL,
  blocked_states = NULL,
  state_labels = list()
)

gw_s2rc(s)

gw_rc2s(rc)

gw_matrix(model, epoch = 1L, what = "states")

gw_plot(
  model,
  epoch = 1L,
  actions = "character",
  states = TRUE,
```

```
  index = FALSE,
  labels = TRUE,
  impossible_actions = FALSE,
  main = NULL,
  cex = 1,
  offset = 0.5,
  lines = TRUE,
  col = hcl.colors(100, "YlOrRd", rev = TRUE),
  blocked_col = "gray20",
  ...
)

gw_plot_transition_graph(
  x,
  remove.loops = TRUE,
  vertex.color = "gray",
  vertex.shape = "square",
  vertex.size = 10,
  vertex.label = NA,
  edge.arrow.size = 0.3,
  margin = 0.2,
  main = NULL,
  ...
)

gw_animate(model, method, n, zlim = NULL, continue = FALSE, ...)

gw_transition_prob(model, action, start.state)

gw_transition_prob_sparse(model, action, start.state)

gw_transition_prob_named(model, action, start.state)

gw_transition_prob_end_state(model, action, start.state, end.state)

gw_maze_MDP(
  dim,
  start,
  goal,
  walls = NULL,
  actions = c("up", "right", "down", "left"),
  goal_reward = 100,
  step_cost = 1,
  restart = FALSE,
  discount = 1,
  horizon = Inf,
  info = NULL,
  normalize = FALSE,
```

```
  name = "Maze"
)

gw_maze_MDPTF(
  dim,
  start,
  goal,
  walls = NULL,
  actions = c("up", "right", "down", "left"),
  goal_reward = 100,
  step_cost = 1,
  discount = 1,
  horizon = Inf,
  info = NULL,
  normalize = FALSE,
  name = "Maze"
)

gw_random_maze(
  dim,
  wall_prob = 0.2,
  start = NULL,
  goal = NULL,
  normalize = FALSE
)

gw_read_maze(file, discount = 1, restart = FALSE, name = "Maze")

gw_path(model, start = NULL, goal = NULL, horizon = NULL)
```

Arguments

dim	vector of length two with the x and y extent of the gridworld.
actions	how to show actions. Options are: simple "character", "unicode" arrows (needs to be supported by the used font), "label" of the action, and "none" to suppress showing the action.
start, goal	start and goal states. If NULL then the states specified in the model are used.
absorbing_states	a vector with state labels for absorbing states.
blocked_states	a vector with state labels for unreachable states. These states will be excluded.
state_labels	a list with labels for states. The element names need to be state names.
s	a state label or a vector of labels.
rc	a vector of length two with the row and column coordinate of a state in the gridworld matrix. A matrix with one state per row can be also supplied.
model, x	a solved gridworld MDP.
epoch	epoch for unconverged finite-horizon solutions.

what — What should be returned in the matrix. Options are: "states", "index", "labels", "values", "actions", "absorbing", and "unreachable".

states — logical; show state names.

index — logical; show the state indices.

labels — logical; show state labels.

impossible_actions — logical; show the value and the action for absorbing states.

main — a main title for the plot. Defaults to the name of the problem.

cex — expansion factor for the action.

offset — move the state labels out of the way (in fractions of a character width).

lines — logical; draw lines to separate states.

col — a colors for the utility values.

blocked_col — a color used for blocked states. Use NA for no color.

... — further arguments are passed on to igraph::plot.igraph().

remove.loops — logical; do not show transitions from a state back to itself.

vertex.color, vertex.shape, vertex.size, vertex.label, edge.arrow.size — see igraph::igraph.plotting for details. Set vertex.label = NULL to show the state labels on the graph.

margin — a single number specifying the margin of the plot. Can be used if the graph does not fit inside the plotting area.

method — an MDP solution method for solve_MDP().

n — number of iterations to animate.

zlim — limits for visualizing the state value.

continue — logical; continue solving a solution.

action, start.state, end.state — parameters for the transition function.

walls — a vector with state labels for walls. Walls will become unreachable states.

goal_reward — reward to transition to the goal state.

step_cost — cost of each action that does not lead to the goal state.

restart — logical; if TRUE then the problem automatically restarts when the agent reaches the goal state.

discount, horizon — MDP discount factor, and horizon.

info — A list with additional information. Has to contain the gridworld dimensions as element dim and can be created using gw_init().

normalize — logical; should the description be normalized for faster access using normalize_MDP().

name — a string to identify the MDP problem.

wall_prob — probability to make a tile a wall.

file — filename for a maze text file.

Details

Gridworlds are implemented with state names `s(row,col)`, where `row` and `col` are locations in the matrix representing the gridworld. The default actions are `"up"`, `"right"`, `"down"`, and `"left"`.

Creating a Gridworld:

`gw_init()` initializes a new gridworld creating a matrix of states with the given dimensions. Other action names can be specified, but they must have the same effects in the same order as above. Blocked states (walls) and absorbing state can be defined. This information can be used to build a custom gridworld MDP. Note that blocked states are removed from the model description using `remove_unreachable_states()`.

Converting Between State Names and Coordinates:

`gw_s2rc()` and `gw_rc2s` help with converting from state names to xy-coordinates and vice versa.

Inspecting Gridworlds:

`gw_matrix()` returns different information (state names, values, actions, etc.) as a matrix. Note that some gridworlds have unreachable states removed. These states will be represented in the matrix as `NA`.

`gw_plot()` plots a gridworld.

`gw_plot_transition_graph()` plots the transition graph using the gridworld matrix as the layout.

`gw_animate()` applies algorithms from `solve_MDP()` iteration by iteration and visualized the state utilities. This helps to understand how the algorithms work.

Gridworld Transition Model:

The transition model is available in several forms:

- `gw_transition_prob()` returns a dense vector for the action and start state.
- `gw_transition_prob_sparse()` returns a sparse vector for the action and start state. Note: creating sparse vectors is very expensive and should only be used for sparse models with a large state space.
- `gw_transition_prob_named()` returns only the non-zero probabilities as a named vector.
- `gw_transition_prob_end_state()` returns a single value for a given action, start and end state. Note: Using this function is very slow since it results in excessive function calls.

Mazes:

`gw_maze_MDP()` helps to easily define maze-like gridworld MDPs. By default, the goal state is absorbing, but with `restart = TRUE`, the agent restarts the problem at the start state every time it reaches the goal and receives the reward. Note that this implies that the goal state itself becomes unreachable.

`gw_read_maze()` reads a maze in text format from a file and converts it into a gridworld MDP.

`gw_path()` checks if a solved gridworld has a policy that leads from the start to the goal. Note this function currently samples only a single path which is an issue with stochastic transitions!

Value

`gw_animate()` returns the final solution invisibly.

`gw_maze_MDP()` returns an MDP object.

`gw_path()` returns a list with the elements `"path"`, `"reward"` and `"solved"`.

See Also

Other gridworld: Cliff_walking, DynaMaze, Maze, Windy_gridworld

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Examples

```
# Defines states, actions and a transition model for a standard gridworld
gw <- gw_init(
  dim = c(7, 7),
  blocked_states = c("s(2,2)", "s(7,3)", "s(3,6)"),
  absorbing_states = "s(4,4)",
  state_labels = list("s(4,4)" = "Black Hole")
)

str(gw)

# display the state labels in the gridworld (states not represented in the
# model are shown as NA)
gw_matrix(gw)
gw_matrix(gw, what = "label")
gw_matrix(gw, what = "absorbing")
gw_matrix(gw, what = "unreachable") # these are actually missing from the model

# a transition function for regular moves in the gridworld is provided
gw_transition_prob(gw, "right", "s(1,1)")
gw_transition_prob_end_state(gw, "right", "s(1,1)", "s(1,2)")

# convert between state names and row/column indices
gw_s2rc("s(1,1)")
gw_rc2s(c(1, 1))

# The information in gw can be used to build a custom MDP.

# We modify the standard transition function so there is a 50% chance that
# you will get sucked into the black hole from the adjacent squares.
trans_black_hole <- function(model,
                             action,
                             start.state,
                             end.state) {
  # states around the black hole
  if (start.state %in% c(
    "s(3,3)", "s(3,4)", "s(3,5)", "s(4,3)", "s(4,5)",
    "s(5,3)", "s(5,4)", "s(5,5)"
  )) {
    if (end.state == "s(4,4)") {
      return(.5 + gw_transition_prob_end_state(model, action, start.state,
                                               end.state) * .5)
    } else {
```

```
      return(gw_transition_prob_end_state(model, action, start.state,
                                          end.state) * .5)
    }
  }

  # use the standard gridworld movement
  gw_transition_prob_end_state(model, action, start.state, end.state)
}

black_hole <- MDP(
  states = gw$states,
  actions = gw$actions,
  transition_prob = trans_black_hole,
  reward = rbind(R_(                        value = +1),
                 R_(end.state = "s(4,4)", value = -100),
                 R_(start.state = "s(4,4)", value = 0)
                 ),
  info = gw$info,
  name = "Black hole"
)

black_hole
black_hole <- normalize_MDP(black_hole)

gw_plot_transition_graph(black_hole)

# solve the problem
sol <- solve_MDP(black_hole, error = 1)
gw_matrix(sol, what = "values")
gw_plot(sol)
# the optimal policy is to fly around, but avoid the black hole.

# Build a Maze: The Dyna Maze from Chapter 8 in the RL book

DynaMaze <- gw_maze_MDP(
  dim = c(6, 9),
  start = "s(3,1)",
  goal = "s(1,9)",
  walls = c(
    "s(2,3)", "s(3,3)", "s(4,3)",
    "s(5,6)",
    "s(1,8)", "s(2,8)", "s(3,8)"
  ),
  restart = TRUE,
  discount = 0.95,
  name = "Dyna Maze",
)
DynaMaze

gw_matrix(DynaMaze)
gw_matrix(DynaMaze, what = "labels")

gw_plot_transition_graph(DynaMaze)
```

```
# Note that the problems resets if the goal state would be reached.

sol <- solve_MDP(DynaMaze, method = "LP:LP")

gw_matrix(sol, what = "values")
gw_matrix(sol, what = "actions")
gw_plot(sol, states = TRUE)

# check if we found a solution
gw_path(sol)

# Read a maze from a text file
#   (X are walls, S is the start and G is the goal)

# some examples are installed with the package
maze_dir <- system.file("mazes", package = "markovDP")
dir(maze_dir)

file.show(file.path(maze_dir, "small_maze.txt"))

maze <- gw_read_maze(file.path(maze_dir, "small_maze.txt"))
maze
gw_matrix(maze, what = "label")
gw_plot(maze)

# Prioritized sweeping is especially effective for larger mazes.
sol <- solve_MDP(maze, method = "DP:GenPS")
sol

gw_plot(sol)
gw_path(sol, horizon = 1000)

# A maze can also be created directly from a character vector
maze <- gw_read_maze(
    textConnection(c("XXXXXX",
                     "XS  GX",
                     "XXXXXX")))
gw_plot(maze)

# Create a small random maze
rand_maze <- gw_random_maze(dim = c(5, 5))
gw_plot(rand_maze)
```

Maze *Steward Russell's 4x3 Maze Gridworld MDP*

Description

The 4x3 maze is described in Chapter 17 of the textbook "Artificial Intelligence: A Modern Approach" (AIMA).

Format

An object of class MDP.

Details

The simple maze has the following layout:

```
  1234           Transition model:
 ######             .8 (action direction)
1#   +#             ^
2# # -#             |
3#S   #          .1 <-|-> .1
 ######
```

We represent the maze states as a gridworld matrix with 3 rows and 4 columns. The states are labeled `s(row, col)` representing the position in the matrix. The # (state `s(2,2)`) in the middle of the maze is an obstruction and not reachable. Rewards are associated with transitions. The default reward (penalty) is -0.04. The start state marked with `S` is `s(3,1)`. Transitioning to `+` (state `s(1,4)`) gives a reward of +1.0, transitioning to `-` (state `s_(2,4)`) has a reward of -1.0. Both these states are absorbing (i.e., terminal) states.

Actions are movements (`up`, `right`, `down`, `left`). The actions are unreliable with a .8 chance to move in the correct direction and a 0.1 chance to instead to move in a perpendicular direction leading to a stochastic transition model.

Note that the problem has reachable terminal states which leads to a proper policy (that is guaranteed to reach a terminal state). This means that the solution also converges without discounting (`discount = 1`).

References

Russell,9 S. J. and Norvig, P. (2020). Artificial Intelligence: A modern approach. 4rd ed.

See Also

Other MDP_examples: `Cliff_walking`, `DynaMaze`, `MDP()`, `Windy_gridworld`

Other gridworld: `Cliff_walking`, `DynaMaze`, `Windy_gridworld`, `gridworld`

Examples

```
# The problem can be loaded using data(Maze).

# Here is the complete problem definition.

# We first look at the state layout
gw_matrix(gw_init(dim = c(3, 4)))

# the wall at s(2,2) is unreachable
gw <- gw_init(dim = c(3, 4),
        start = "s(3,1)",
        goal = "s(1,4)",
```

```
        absorbing_states = c("s(1,4)", "s(2,4)"),
        blocked_states = "s(2,2)",
        state_labels = list(
            "s(3,1)" = "Start",
            "s(2,4)" = "-1",
            "s(1,4)" = "Goal: +1")
)
gw_matrix(gw)
gw_matrix(gw, what = "index")
gw_matrix(gw, what = "labels")

# gw_init has created the following information
str(gw)

# the transition function is stochastic so we cannot use the standard
# gridworld function provided in gw$transition_prob() and we
# have to replace it
P <- function(model, action, start.state) {
  action <- match.arg(action, choices = A(model))

  P <- structure(numeric(length(S(model))), names = S(model))

  # absorbing states
  if (start.state %in% model$info$absorbing_states) {
    P[start.state] <- 1
    return(P)
  }

  if (action %in% c("up", "down")) {
    error_direction <- c("right", "left")
  } else {
    error_direction <- c("up", "down")
  }

  rc <- gw_s2rc(start.state)
  delta <- list(
    up = c(-1, 0),
    down = c(+1, 0),
    right = c(0, +1),
    left = c(0, -1)
  )

  # there are 3 directions. For blocked directions, stay in place
  # 1) action works .8
  rc_new <- gw_rc2s(rc + delta[[action]])
  if (rc_new %in% S(model))
    P[rc_new] <- .8
  else
    P[start.state] <- .8

  # 2) off to the right .1
  rc_new <- gw_rc2s(rc + delta[[error_direction[1]]])
  if (rc_new %in% S(model))
```

```
    P[rc_new] <- .1
  else
    P[start.state] <-  P[start.state] + .1

  # 3) off to the left .1
  rc_new <- gw_rc2s(rc + delta[[error_direction[2]]])
  if (rc_new %in% S(model))
    P[rc_new] <- .1
  else
    P[start.state] <-  P[start.state] + .1

  P
  }

P(gw, "up", "s(3,1)")

R <- rbind(
  R_(                          value = -0.04),
  R_(end.state = "s(2,4)",     value = -1 - 0.04),
  R_(end.state = "s(1,4)",     value = +1 - 0.04),
  R_(start.state = "s(2,4)",   value = 0),
  R_(start.state = "s(1,4)",   value = 0)
)


Maze <- MDP(
  name = "Stuart Russell's 3x4 Maze",
  discount = 1,
  horizon = Inf,
  states = gw$states,
  actions = gw$actions,
  start = "s(3,1)",
  transition_prob = P,
  reward = R,
  info = gw$info
)

Maze

str(Maze)

gw_matrix(Maze)
gw_matrix(Maze, what = "labels")
gw_plot(Maze)

# find absorbing (terminal) states
absorbing_states(Maze)

maze_solved <- solve_MDP(Maze)
policy(maze_solved)

gw_matrix(maze_solved, what = "values")
gw_matrix(maze_solved, what = "actions")
```

```
gw_plot(maze_solved)
```

MDP *Define an MDP Problem*

Description

Defines all the elements of a discrete-time finite state-space MDP problem.

Usage

```
MDP(
  states,
  actions,
  transition_prob,
  reward,
  discount = 0.9,
  horizon = Inf,
  start = "uniform",
  info = NULL,
  name = NA
)

S(model)

A(model)

is_solved_MDP(model, policy = TRUE, approx = FALSE, stop = FALSE)

is_converged_MDP(model, stop = FALSE)

P_(action = NA, start.state = NA, end.state = NA, probability)

R_(action = NA, start.state = NA, end.state = NA, value)
```

Arguments

states	a character vector specifying the names of the states.
actions	a character vector specifying the names of the available actions.
transition_prob	Specifies the transition probabilities between states.
reward	Specifies the rewards dependent on action and states.
discount	numeric; discount rate between 0 and 1.
horizon	numeric; Number of epochs. `Inf` specifies an infinite horizon.
start	Specifies in which state the MDP starts.

info	A list with additional information.
name	a string to identify the MDP problem.
model	an MDP object.
policy	logical; solution is an explicit policy.
approx	logical; solution is an approximation function.
stop	logical; stop with an error.
action	action as a action label or integer. The value NA matches any action.
start.state, end.state	state as a state label or an integer. The value NA matches any state.
probability, value	Values used in the helper functions P_() and R_().

Details

Markov decision processes (MDPs) are discrete-time stochastic control process. We implement here MDPs with a finite state space. MDP() defines all the element of an MDP problem including the discount rate, the set of states, the set of actions,the transition probabilities, and the rewards.

In the following we use the following notation. The MDP is a 5-duple:

(S, A, P, R, γ).

S is the set of states; A is the set of actions; P are the conditional transition probabilities between states; R is the reward function; and γ is the discount factor. We will use lower case letters to represent a member of a set, e.g., s is a specific state. To refer to the size of a set we will use cardinality, e.g., the number of actions is $|A|$.

Names used for mathematical symbols in code:

- S, s, s': 'states', start.state', 'end.state'
- A, a: 'actions', 'action'

State names and actions can be specified as strings or index numbers (e.g., start.state can be specified as the index of the state in states). For the specification as data.frames below, NA can be used to mean any start.state, end.state or action.

Specification of transition model: $P(s'|s, a)$:

Transition probability to transition to state s' from given state s and action a. The transition probabilities can be specified in the following ways:

- A data.frame with columns exactly like the arguments of P_(). You can use rbind() with helper function P_() to create this data frame. Probabilities can be specified multiple times and the definition that appears last in the data.frame will take affect.
- A named list of matrices, one for each action. Each matrix is square with rows representing start states s and columns representing end states s'. Instead of a matrix, also the strings 'identity' or 'uniform' can be specified.
- A function with the following arguments:
  - A function with the argument list model, action, start.state, end.state which returns a single transition probability.

- A function with the argument list `model`, `action`, `start.state` which returns a transition probability vector for all end states. This vector can be dense, a Matrix::sparseVector or a named vector only containing the non-zero probabilities named by the corresponding end state.

The arguments `action`, `start.state`, and `end.state` will be always called with the state names as a character vectors of length 1.

Specification of the reward function: $R(a, s, s')$:

The reward function can be specified in the following ways:

- A data frame with columns named exactly like the arguments of `R_()`. You can use `rbind()` with helper function `R_()` to create this data frame. Rewards can be specified multiple times and the definition that appears last in the data.frame will take affect.
- A named list of matrices, one for each action. Each matrix is square with rows representing start states s and columns representing end states s'.
- A function following the same rules as for transition probabilities.

To avoid overflow problems with rewards, reward values should stay well within the range of `[-1e10, +1e10]`. `-Inf` can be used as the reward for unavailable actions and will be translated into a large negative reward for solvers that only support finite reward values.

Specification of the Start State:

The start state of the agent can be a single state or a distribution over the states. The start state definition is used as the default when the reward is calculated by `reward()` and for sampling with `sample_MDP()`.

Options to specify the start state are:

- A string specifying the name of a single starting state.
- An integer in the range 1 to n to specify the index of a single starting state.
- The string `"uniform"` where the start state is chosen using a uniform distribution over all states.
- A probability distribution over the states. That is, a vector of $|S|$ probabilities, that add up to 1.

The default state state is a uniform distribution over all states.

Accessing Elements of the MDP:

The convenience functions `S()` and `A()` return the set of states and actions.

See accessors for accessing transition probabilities, rewards, and the start state distribution.

Value

The function returns an object of class MDP which is list with the model specification. `solve_MDP()` reads the object and adds a list element called `'solution'`.

Author(s)

Michael Hahsler

See Also

Other MDP: Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other MDP_examples: Cliff_walking, DynaMaze, Maze, Windy_gridworld

Examples

```
# simple MDP example
#
# states:    s1 s2 s3 s4
# transitions: forward moves -> and backward moves <-
# start: s1
# reward: s1, s2, s4 = 0 and s3 = 1

car <- MDP(
  states = c("s1", "s2", "s3", "s4"),
  actions = c("forward", "back", "stop"),
  transition <- list(
    forward = rbind(c(0, 1, 0, 0),
                    c(0, 0, 1, 0),
                    c(0, 0, 0, 1),
                    c(0, 0, 0, 1)),
    back =    rbind(c(1, 0, 0, 0),
                    c(1, 0, 0, 0),
                    c(0, 1, 0, 0),
                    c(0, 0, 1, 0)),
    stop = "identity"
  ),
  reward = rbind(
    R_(value = 0),
    R_(end.state = "s3", value = 1)
  ),
  discount = 0.9,
  start = "s1",
  name = "Simple Car MDP"
)

car

# internal representation
str(car)

# accessing elements
S(car)
A(car)
start_vector(car, sparse = "states")
transition_matrix(car)
transition_matrix(car, sparse = TRUE)
reward_matrix(car)
```

```
reward_matrix(car, sparse = TRUE)

sol <- solve_MDP(car)
sol

policy(sol)
```

MDPTF *Define an MDP as an Agent Environment*

Description

Defines a discrete-time agent environment. The environment's MDP is defined via a transition function and states and the transition probabilities are not directly specified.

Usage

```
MDPTF(
  actions,
  transition_func,
  start,
  states = NULL,
  absorbing_states = NULL,
  discount = 0.9,
  horizon = Inf,
  info = NULL,
  name = NA
)
```

Arguments

actions	a character vector specifying the names of the available actions.
transition_func	A transition function receiving the current state features and returning the reward and the next state features.
start	Specifies in which state the MDP starts.
states	Optional state labels.
absorbing_states	a single state or a list with absorbing states.
discount	numeric; discount rate between 0 and 1.
horizon	numeric; Number of epochs.
info	A list with additional information.
name	a string to identify the MDP problem.

Details

Defines a discrete-time agent environment. The environment is defined via a transition function using state features (i.e., using a factored state representation). In comparison to the MDP, where transition probabilities are directly specified and potentially used by solvers, the MDPTF has no specification of transition probabilities and the potentially infinite set of states is unknown.

To represent states, a factored state representation as a **row** vector for a single state or a matrix with row vectors for a set of states are used. State labels are constructed in the form `s(feature1, feature2, ...)`. Conversion between the factored representation and state labels is available in `state2features()` and `features2state()`. Since the state set is not directly represented, **state ids are cannot be used!**

Reinforcement learning algorithms with approximation can be used to solve these problems. See: `solve_MDP_APPROX()`.

Value

The function returns an object of class MDPTF which is list with the model specification.

Author(s)

Michael Hahsler

See Also

Other MDPTF: `absorbing_states()`, `act()`, `sample_MDP.MDPTF()`, `solve_MDP()`, `solve_MDP_APPROX()`, `start()`

Examples

```
# Define a simple 5x5 maze without walls

transition_func <- function(model, state, action) {
  if (all(state == s(5, 5)))
    return(list(reward = 0, state_prime = state))

  action <- normalize_action_label(action, model)

  sp <- state + switch(action,
    up =   c( -1, 0),
    down = c( +1, 0),
    left = c(  0,-1),
    right = c( 0,+1)
  )
  r <- -1

  # check bounds
  if (any(sp < 1) || any(sp > 5)) {
      sp <- state
   }

  # goal
```

```
  if (all(sp == s(5, 5)))
    r <- 100

  return(list(reward = r, state_prime = sp))
}

m <- MDPTF(actions = c("up", "right", "down", "left"),
          transition_func,
          start = s(1,1),
          absorbing_states = s(5, 5),
          name = "5x5 Maze")
m

act(m, s(1,1), "down")

# Reach the goal: reward = 100
act(m, s(5,4), "right")

# Illegal action: no movement and reward = -1
act(m, s(1,1), "up")

# Absorbing state: no movement and 0 reward
act(m, s(5,5), "up")

# Example: Solve using Linear Feature Approximation

# the same maze can be created with this helper and then gridworld helper
#    functions can be used
m <- gw_maze_MDPTF(c(5,5), start = "s(1,1)", goal = "s(5,5)")

m_approx <- add_linear_approx_Q_function(m)
sol <- solve_MDP_APPROX(m_approx, horizon = 1000, n = 10,
          alpha = 0.01, epsilon = 0.8, verbose = FALSE)
gw_plot(sol)

approx_greedy_action(sol, s(1,1))
approx_greedy_action(sol, s(5,4))
# to use the greedy_action interface, the state label has to be used
greedy_action(sol, "s(1,1)")

approx_Q_value(sol, s(1,1))
approx_Q_value(sol, s(5,4))

# Example: Solve using order-2 Fourier Basis Features
fourier_basis_trans <- function(x) {
  x <- x / 10
  cs <- expand.grid(0:2, 0:2)
  apply(cs, MARGIN = 1, FUN = function(c) cos(pi * x %*% c))
}

m_approx <- add_linear_approx_Q_function(m, transformation = fourier_basis_trans)
sol <- solve_MDP_APPROX(m_approx, horizon = 1000, n = 10,
          alpha = 0.01, epsilon = 0.7, verbose = FALSE)
```

```
gw_plot(sol)
sol <- solve_MDP_APPROX(sol, horizon = 1000, n = 100,
          alpha = 0.01, epsilon = 0.05, verbose = FALSE, continue = TRUE)
gw_plot(sol)
```

`policy` *Extract, Create Add a Policy to a Model*

Description

Extracts the policy from a solved model or create a policy. All policies are deterministic.

Usage

```
policy(model, epoch = NULL, drop = TRUE)

add_policy(model, policy)

random_policy(
  model,
  prob = NULL,
  estimate_V = FALSE,
  only_available_actions = FALSE,
  ...
)

manual_policy(model, actions, V = NULL, estimate_V = FALSE)

induced_transition_matrix(model, policy = NULL, epoch = 1L, sparse = FALSE)

induced_reward_matrix(model, policy = NULL, epoch = 1L)
```

Arguments

`model`	A solved MDP object.
`epoch`	return the policy of the given epoch. `NULL` returns a list with elements for each epoch.
`drop`	logical; drop the list for converged, epoch-independent policies.
`policy`	a policy data.frame.
`prob`	probability vector for random actions for `random_policy()`. a logical indicating if action probabilities should be returned for `greedy_action()`.
`estimate_V`	logical; estimate the value function using `policy_evaluation()`?
`only_available_actions`	logical; only sample from available actions? (see `available_actions()` for details)

...	is passed on to available_actions().
actions	a vector with the action (either the action label or the numeric id) for each state.
V	a vector representing the value function for the policy. If TRUE, then the it is estimated using policy_evaluation().
sparse	logical; should a sparse transition matrix be returned?

Details

policy() extracts the (deterministic) policy from a solved MDP in the form of a a data.frame with columns for:

- state: The state.
- V: The state values if the policy is followed.
- action: The prescribed action.

For unconverged, finite-horizon problems, the solution is a policy for each epoch. This is returned as a list of data.frames.

add_policy() adds a policy to an existing MDP object.

random_policy() and manual_policy() construct new policies.

induced_transition_matrix() returns the single transition matrix which follows the actions specified in a policy.

Value

- policy(), random_policy() and manual_policy() return a data.frame containing the policy. If drop = FALSE then the policy is returned as a list with the policy for each epoch.
- add_policy() returns an MDP object.
- induced_transition_matrix returns a single transition matrix.

The model description with the added policy.

Author(s)

Michael Hahsler

See Also

Other policy: Q_values(), action(), bellman_update(), greedy_action(), policy_evaluation(), regret(), reward(), value_function(), visit_probability()

Examples

```
data("Maze")

sol <- solve_MDP(Maze)
sol

## policy with value function and optimal action.
```

```
policy(sol)
plot_value_function(sol)
gw_plot(sol)

induced_transition_matrix(sol)

## create a random policy
pi_random <- random_policy(Maze, estimate_V = TRUE)
pi_random

gw_plot(add_policy(Maze, pi_random))

## create a manual policy (go up and in some squares to the right)
acts <- rep("up", times = length(Maze$states))
names(acts) <- Maze$states
acts[c("s(1,1)", "s(1,2)", "s(1,3)")] <- "right"
acts

pi_manual <- manual_policy(Maze, acts, estimate_V = TRUE)
pi_manual

gw_plot(add_policy(Maze, pi_manual))

# Transition matrix induced by the policy
induced_transition_matrix(Maze, pi_manual, sparse = TRUE)

## Finite horizon (we use incremental pruning because grid does not converge)
sol <- solve_MDP(model = Maze, horizon = 3)
sol

policy(sol)
gw_plot(sol, epoch = 1)
gw_plot(sol, epoch = 2)
gw_plot(sol, epoch = 3)
```

`policy_evaluation` *Policy Evaluation*

Description

Estimate the value function for a policy applied to a model by repeatedly applying the Bellman operator.

Usage

```
policy_evaluation(
  model,
  pi = NULL,
  V = NULL,
  k_backups = 1000L,
```

```
  theta = 0.001,
  progress = TRUE,
  verbose = FALSE
)

policy_evaluation_LP(model, pi = NULL, inf = 1000, verbose = FALSE, ...)
```

Arguments

model	an MDP problem specification.
pi	a policy as a data.frame with at least columns for states and action. If NULL, then the policy in model is used.
V	a vector with estimated state values representing a value function. If model is a solved model, then the state values are taken from the solution.
k_backups	number of look ahead steps used for approximate policy evaluation used by the policy iteration method. Set k_backups to Inf to only use θ as the stopping criterion.
theta	stop when the largest state Bellman error ($\delta = V_{k+1} - V$) is less than θ.
progress	logical; show a progress bar with estimated time for completion.
verbose	logical; should progress and approximation errors be printed.
inf	value used to replace infinity for lpSolve::lp().
...	further arguments are ignored

Details

The value function for a policy can be estimated (called policy evaluation) by repeatedly applying the Bellman operator

$$v \leftarrow B_\pi(v)$$

till convergence.

In each iteration, all state values are updated. In this implementation updating is stopped when the largest state Bellman error is below a threshold.

$$||v_{k+1} - v_k||_\infty < \theta.$$

Or if k_backups iterations have been completed.

The LP implementation only works for infinite-horizon problems with a discount factor <1. It solves the following LP:

$$\min \sum_{s \in S} v(s)$$

s.t.

$$v(s) \geq \sum_{s' \in S} p(s'|s, \pi(s))[r(s, \pi(s), s') + \gamma v(s')], \ \forall s \in S$$

Value

a vector with (approximate) state values (U).

Author(s)

Michael Hahsler

References

Sutton, R. S., Barto, A. G. (2020). Reinforcement Learning: An Introduction. Second edition. The MIT Press.

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other policy: Q_values(), action(), bellman_update(), greedy_action(), policy(), regret(), reward(), value_function(), visit_probability()

Examples

```
data(Maze)
Maze

# create several policies:
# 1. optimal policy using value iteration
maze_solved <- solve_MDP(Maze, method = "DP:VI")
pi_opt <- policy(maze_solved)
pi_opt

# 2. a manual policy (go up and in some squares to the right)
acts <- rep("up", times = length(Maze$states))
names(acts) <- Maze$states
acts[c("s(1,1)", "s(1,2)", "s(1,3)")] <- "right"
pi_manual <- manual_policy(Maze, acts)
pi_manual

# 3. a random policy
set.seed(1234)
pi_random <- random_policy(Maze, prob = c(up = .7, right = .1, down = .1, left = 0.1))
pi_random

# 4. an improved policy based on one policy evaluation and
#   policy improvement step.
V <- policy_evaluation(Maze, pi_random)
Q <- Q_values(Maze, V)
pi_greedy <- greedy_policy(Q)
pi_greedy

#' compare the approx. value functions for the policies (we restrict
```

```
#'    the number of backups for the random policy since it may not converge)
rbind(
  random = policy_evaluation(Maze, pi_random, k_backups = 100),
  manual = policy_evaluation(Maze, pi_manual),
  greedy = policy_evaluation(Maze, pi_greedy),
  optimal = policy_evaluation(Maze, pi_opt)
)
```

`Q_values` *Q-Values*

Description

Several useful functions to deal with Q-values (action values) which map each state/action pair to a utility value.

Usage

```
Q_values(model, V = NULL, state = NULL)

Q_zero(model, value = 0)

Q_random(model, min = 1e-06, max = 1)
```

Arguments

`model`	an MDP problem specification.
`V`	the state values. If `model` is a solved model, then the state values are taken from the solution.
`state`	specify the state. If `NULL` then the Q-values for all states is returned as a matrix.
`value`	value to initialize the Q-value matrix. Default is 0.
`min`, `max`	range of the random values

Details

Implemented functions are:

- `Q_values()` gets the Q-values from the model. If the value function `V` is specified or if the policy of a solved model contains a value function, then the Q-values are approximated using the Bellman optimality equation:

$$q_*(s,a) = \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v_*(s')]$$

  Exact Q values are calculated if $v = v_*$, the optimal value function, otherwise we get an approximation that might not be consistent with v or the implied policy. Q values can be used as the input for several other functions.

  For solvers, that directly calculate Q-values or an approximate Q-function, then the solver's values are returned.
- `Q_zero()` and `Q_random()` create initial Q value matrices for algorithms.

Value

`Q_values()` returns a state by action matrix specifying the Q-function, i.e., the action value for executing each action in each state. The Q-values are calculated from the value function (U) and the transition model.

`Q_zero()` and `Q_random` return a matrix with q-values.

Author(s)

Michael Hahsler

References

Sutton, R. S., Barto, A. G. (2020). Reinforcement Learning: An Introduction. Second edition. The MIT Press.

See Also

Other MDP: `MDP()`, `absorbing_states()`, `act()`, `available_actions()`, `bellman_update()`, `convergence_horizon()`, `greedy_action()`, `gridworld`, `policy_evaluation()`, `regret()`, `sample_MDP()`, `solve_MDP()`, `start()`, `transition_graph()`, `transition_matrix()`, `unreachable_states()`, `value_function()`

Other policy: `action()`, `bellman_update()`, `greedy_action()`, `policy()`, `policy_evaluation()`, `regret()`, `reward()`, `value_function()`, `visit_probability()`

Examples

```
data(Maze)
Maze

# create a random policy and calculate q-values
pi_random <- random_policy(Maze)
pi_random

V <- policy_evaluation(Maze, pi_random)
V

# calculate Q values
Q <- Q_values(Maze, V)
Q
```

---

regret *Regret of a Policy and Related Measures*

Description

Calculates the regret and related measures for a policy relative to a benchmark policy.

Usage

```
regret(policy, benchmark, start = NULL, relative = FALSE, ...)

action_discrepancy(policy, benchmark, weighted = FALSE, proportion = FALSE)

value_error(policy, benchmark, type = "RMSVE", weighted = FALSE)
```

Arguments

policy	a solved MDP containing the policy to calculate the regret for.
benchmark	a solved MDP with the (optimal) policy. Regret is calculated relative to this policy.
start	start state distribution. If NULL then the start state of the benchmark is used.
relative	logical; should the relative regret (regret divided by the reward of the benchmark) be calculated?
...	further arguments are passed on to reward().
weighted	logical; should mismatched actions or state value errors be weighted by the state visit probability for the benchmark? Rarely or never visited states will have now less influence on the measure.
proportion	logical; should the action discrepancy be reported as a proportion of states with a different action.
type	type of error root mean square value error ("RMSVE"), mean square value error ("MSVE"), mean absolute value error ("MAVE"), absolute value error vector ("AVE"), value error vector ("VE").

Details

Regret:

Regret for a policy π is defined as

$$v_\pi(s_0) - v_*(s_0),$$

where $v_\pi(s_0)$ represents the expected long-term state value for following policy π and the starting in state s_0 (or a start distribution). The relative regret is calculated as

$$\frac{v_\pi(s_0) - v_*(s_0)}{v_*(s_0)}.$$

Note that for regret, usually the optimal policy π^* is used as the benchmark. Since the optimal policy may not be known, regret relative to the best known policy can be used.

Regret is only valid with converged value functions. This means that either the solver has converged, or the value function was estimated for the policy using converged policy_evaluation().

Action Discrepancy:

The action discrepancy measures the difference between two policies as the number of states for which the prescribed action in the policies differs. Often, a policy is compared to the best known policy called the benchmark policy.

Some times two actions are equivalent (have the same q-value) and the algorithm breaks the tie randomly. The implementation accounts for this case.

The action discrepancy can be calculated as a proportion of different actions or be weighted by the state visit probability given the benchmark policy. Both weighted and proportional action discrepancy is scaled in $[0, 1]$.

Root Mean Squared Value Error:

The root mean value error

$$\sqrt{\text{VE}} = \sqrt{||v_\pi - v_*||^2}$$

is the sum of the squared differences of state values between a solution's value function and the optimal value function. For v_*, the value function of the benchmark solution is used. Related measures like MSVE (means squared value error), MAVE (means absolute value error), AVE (absolute value error) and VE (value error) are also provided.

The error can also be weighted by the state visit probability given the benchmark policy. This results in the expected error with respect to the state visit distribution of the benchmark policy. This may be important to evaluate methods that focus only on estimating the value function for states that are actually visited using the policy.

Value

- `regret()` returns the regret as a difference of expected long-term rewards.
- `action_discrepancy()` returns the number or proportion of diverging actions.
- `mean_value_error()` returns the mean squared or absolute difference in the value function.

Author(s)

Michael Hahsler

See Also

Other MDP: `MDP()`, `Q_values()`, `absorbing_states()`, `act()`, `available_actions()`, `bellman_update()`, `convergence_horizon()`, `greedy_action()`, `gridworld`, `policy_evaluation()`, `sample_MDP()`, `solve_MDP()`, `start()`, `transition_graph()`, `transition_matrix()`, `unreachable_states()`, `value_function()`

Other policy: `Q_values()`, `action()`, `bellman_update()`, `greedy_action()`, `policy()`, `policy_evaluation()`, `reward()`, `value_function()`, `visit_probability()`

Examples

```
data(Maze)

sol_optimal <- solve_MDP(Maze)

# a manual policy (go up and in some squares to the right)
acts <- rep("up", times = length(Maze$states))
names(acts) <- Maze$states
acts[c("s(1,1)", "s(1,2)", "s(1,3)")] <- "right"
```

```
sol_manual <- add_policy(Maze, manual_policy(Maze, acts, estimate_V = TRUE))

# compare the policies side-by-side
cbind(opt = policy(sol_optimal), manual = policy(sol_manual))

# the regret is very small. It is about 4.8% of the optimal reward
regret(sol_manual, benchmark = sol_optimal)
regret(sol_manual, benchmark = sol_optimal, relative = TRUE)

# The number of different actions (excluding equivalent actions) is 3.
# This about 27% of the actions in the policy.
action_discrepancy(sol_manual, benchmark = sol_optimal)
action_discrepancy(sol_manual, benchmark = sol_optimal, proportion = TRUE)

# Weighted by the probability that a state will be visited shows that
# only 2.3% of the time a different action would be used.
action_discrepancy(sol_manual, benchmark = sol_optimal, weighted = TRUE)

value_error(sol_manual, benchmark = sol_optimal, type = "VE")
value_error(sol_manual, benchmark = sol_optimal, type = "MAVE")
value_error(sol_manual, benchmark = sol_optimal, type = "RMSVE")

# Weighting shows that the expected MAVE (expectation taken over the
# benchmark policy) is rather small.
value_error(sol_manual, benchmark = sol_optimal, type = "MAVE", weighted = TRUE)
```

`reward` *Calculate the Expected Reward of a Policy*

Description

This function calculates the expected total reward for an MDP policy given a start state (distribution). The value is calculated using the value function stored in the MDP solution.

Usage

```
reward(model, ...)

## S3 method for class 'MDP'
reward(model, start = NULL, method = "solution", ...)
```

Arguments

`model`	a solved MDP object.
`...`	further arguments are passed on to `policy_evaluation()` or `sample_MDP()`.
`start`	specification of the current state (see argument start in MDP for details). By default the start state defined in the model as start is used. Multiple states can be specified as rows in a matrix.

method "solution" uses the converged value function stored in the solved model, "policy_evaluation" estimates the value function, and "sample" calculates the average reward by sampling episodes from the model.

Details

The reward is typically calculated using the value function of the solution. If these are not available, then sample_MDP() is used instead with a warning.

Value

reward() returns a vector of reward values, one for each belief if a matrix is specified.

state start state to calculate the reward for. if NULL then the start state of model is used.

Author(s)

Michael Hahsler

See Also

Other policy: Q_values(), action(), bellman_update(), greedy_action(), policy(), policy_evaluation(), regret(), value_function(), visit_probability()

Examples

```
data("Maze")
Maze
gw_matrix(Maze)

sol <- solve_MDP(Maze)
policy(sol)

# reward for the start state s(3,1) specified in the model
reward(sol)

# reward for starting next to the goal at s(1,3)
reward(sol, start = "s(1,3)")

# expected reward when we start from a random state as returned from the solver
reward(sol, start = "uniform")

# estimate the reward using sampling following the policy
reward(sol, method = "sample", start = "uniform", n = 10000, horizon = 1000)
```

round_stochastic *Round a stochastic vector or a row-stochastic matrix*

Description

Rounds a vector such that the sum of 1 is preserved. Rounds a matrix such that each row sum up to 1. One entry is adjusted after rounding such that the rounding error is the smallest.

Usage

```
round_stochastic(x, digits = 7)
```

Arguments

x	a stochastic vector or a row-stochastic matrix.
digits	number of digits for rounding.

Value

The rounded vector or matrix.

See Also

round

Examples

```
# regular rounding would not sum up to 1
x <- c(0.333, 0.334, 0.333)

round_stochastic(x)
round_stochastic(x, digits = 2)
round_stochastic(x, digits = 1)
round_stochastic(x, digits = 0)


# round a stochastic matrix
m <- matrix(runif(15), ncol = 3)
m <- sweep(m, 1, rowSums(m), "/")

m
round_stochastic(m, digits = 2)
round_stochastic(m, digits = 1)
round_stochastic(m, digits = 0)
```

sample_MDP *Sample Trajectories from an MDP*

Description

Sample trajectories through an MDP. The start state for each trajectory is chosen using the start definition in the model. Actions are chosen randomly of using an epsilon-greedy policy.

Usage

```
sample_MDP(model, n, ...)

## S3 method for class 'MDP'
sample_MDP(
  model,
  n,
  start = NULL,
  horizon = NULL,
  epsilon = NULL,
  exploring_starts = FALSE,
  delta_horizon = 0.001,
  trajectories = FALSE,
  engine = NULL,
  progress = TRUE,
  verbose = FALSE,
  ...
)
```

Arguments

model	an MDP model.
n	number of trajectories.
...	further arguments are ignored.
start	probability distribution over the states for choosing the starting states for the trajectories. Defaults to "uniform".
horizon	epochs end once an absorbing state is reached or after the maximal number of epochs specified via horizon. If NULL then the horizon for the model is used.
epsilon	the probability of random actions for using an epsilon-greedy policy. Default for solved models is 0 and for unsolved model 1.
exploring_starts	logical; randomly sample a start/action combination to start the episode from.
delta_horizon	precision used to determine the horizon for infinite-horizon problems.
trajectories	logical; return the complete trajectories.

engine	'cpp' or 'r' to perform simulation using a faster C++ or a native R implementation NULL uses the C++ implementation unless the transition model or the reward are specified as R functions (which are slow in C++).
progress	show a progress bar?
verbose	report used parameters

Details

The default is a faster C++ implementation (engine = 'cpp'). A native R implementation is available (engine = 'r').

Both implementations support parallel execution using the package **foreach**. To enable parallel execution, a parallel backend like **doparallel** needs to be available needs to be registered (see doParallel::registerDoParallel()). Note that small samples are slower using parallelization. Therefore, C++ simulations with n * horizon less than 100,000 are always executed using a single worker.

Value

A list with elements:

- avg_reward: The average discounted reward.
- reward: Reward for each trajectory.
- action_cnt: Action counts.
- state_cnt: State counts.
- trajectories: A data.frame with the trajectories. Each row contains the episode id, the time step, the state s, the chosen action a, the reward r, and the next state s_prime. Trajectories are only returned for trajectories = TRUE.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Examples

```
# enable parallel simulation
# doParallel::registerDoParallel()

data(Maze)

# solve the MDP for 5 epochs and no discounting
sol <- solve_MDP(Maze, discount = 1)
sol
```

```
# V in the policy is and estimate of the state values when following the optimal policy.
policy(sol)
gw_matrix(sol, what = "action")

## Example 1: simulate 100 trajectories following the policy,
#             only the final belief state is returned
sim <- sample_MDP(sol, n = 100, horizon = 10, verbose = TRUE)
sim

# Note that all simulations for this model start at s_1 and that the simulated avg. reward
# is therefore an estimate to the value function for the start state s_1.
policy(sol)[1, ]

# Calculate proportion of actions taken in the simulation
round_stochastic(sim$action_cnt / sum(sim$action_cnt), 2)

# reward distribution
hist(sim$reward)

## Example 2: simulate starting following a uniform distribution over all
#             states and return all trajectories
sim <- sample_MDP(sol,
  n = 100, start = "uniform", horizon = 10,
  trajectories = TRUE
)
head(sim$trajectories)

# how often was each state visited?
table(sim$trajectories$s)
```

`sample_MDP.MDPTF` *Sample Trajectories from an MDPTF*

Description

Sample trajectories through using a MDPTF.

Usage

```
## S3 method for class 'MDPTF'
sample_MDP(
  model,
  n,
  start = NULL,
  horizon = NULL,
  epsilon = NULL,
  exploring_starts = FALSE,
  trajectories = FALSE,
```

```
  progress = TRUE,
  verbose = FALSE,
  ...
)
```

Arguments

model	an MDPTF model.
n	number of trajectories.
start	start state.
horizon	epochs end once an absorbing state is reached or after the maximal number of epochs specified via horizon. If NULL then the horizon for the model is used.
epsilon	the probability of random actions for using an epsilon-greedy policy. Default for solved models is 0 and for unsolved model 1.
exploring_starts	logical; randomly sample a start/action combination to start the episode from.
trajectories	logical; return the complete trajectories.
progress	show a progress bar?
verbose	report used parameters
...	further arguments are ignored.

Value

A list with elements:

- avg_reward: The average discounted reward.
- reward: Reward for each trajectory.
- trajectories: A data.frame with the trajectories. Each row contains the episode id, the time step, the state s, the chosen action a, the reward r, and the next state s_prime. Trajectories are only returned for trajectories = TRUE.

Author(s)

Michael Hahsler

See Also

Other MDPTF: MDPTF(), absorbing_states(), act(), solve_MDP(), solve_MDP_APPROX(), start()

Examples

```
# enable parallel simulation
# doParallel::registerDoParallel()

# Create a simple maze with the layout:
# XXXXXX
# XSX  X
```

```
# X    X
# X    X
# X  XGX
# XXXXXX

model <- gw_maze_MDPTF(
           dim = s(4, 4),
           start = s(1, 1),
           goal = s(4, 4),
           walls = rbind(s(1, 2), s(4, 3)),
           discount = 0.95,
           name = "Simple Maze"
       )
model
gw_plot(model)

sim <- sample_MDP(model, horizon = 500, n = 1,
                  verbose = TRUE, trajectories = TRUE)
sim

# sample from a solved MDPTF by following the policy
model <- add_linear_approx_Q_function(model,
               transformation = transformation_fourier(
                                    min = c(0, 0),
                                    max = c(4,4),
                                    order = 2))
sol <- solve_MDP(model, horizon = 1000, n = 100, alpha = 0.01, epsilon = .1)
gw_plot(sol)

sim <- sample_MDP(sol, horizon = 500, n = 1,
                  verbose = TRUE, trajectories = TRUE)
sim
```

`solve_MDP` *Solve an MDP Problem*

Description

Implementation of value iteration, modified policy iteration and other methods based on reinforcement learning techniques to solve finite state space MDPs.

Usage

```
solve_MDP(model, ...)

## S3 method for class 'MDP'
solve_MDP(
  model,
  method = "DP:VI",
  horizon = NULL,
```

```
  discount = NULL,
  ...,
  matrix = TRUE,
  continue = FALSE,
  verbose = FALSE,
  progress = !verbose
)

## S3 method for class 'MDPTF'
solve_MDP(
  model,
  method = "APPROX:semi_gradient_sarsa",
  horizon = NULL,
  discount = NULL,
  ...,
  matrix = TRUE,
  continue = FALSE,
  verbose = FALSE,
  progress = !verbose
)
```

Arguments

model	an MDP problem specification.
...	further parameters are passed on to the solver function.
method	string; Composed of the algorithm family abbreviation and the algorithm separated by :. The algorithm families can be found in the See Also section under "Other solvers". The family abbreviation follows solve_MDP_ in the function name.
horizon	an integer with the number of epochs for problems with a finite planning horizon. If set to Inf, the algorithm continues running iterations till it converges to the infinite horizon solution. If NULL, then the horizon specified in model will be used.
discount	discount factor in range $(0, 1]$. If NULL, then the discount factor specified in model will be used.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.
progress	logical; show a progress bar with estimated time for completion.

Details

Several solvers are available. Note that some solvers are only implemented for finite-horizon problems.

Most solvers can be interrupted using Esc/CTRL-C and will return the current solution. Solving can be continued by calling `solve_MDP` with the partial solution as the model and the parameter `continue = TRUE`. This method can also be used to reduce parameters like `alpha` or `epsilon` (see Q-learning in the Examples section).

A list of available solvers can be found in the See Also section under "Other solvers".

While MDP model contain an explicit specification of the state space, the transition probabilities and the reward structure, MDPTF only contains a transition function. This means that only a small subset of solvers can be used for MDPTFs. This currently includes only includes the solvers in `solve_MDP_APPROX()`.

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

Author(s)

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References

Russell, Stuart J., and Peter Norvig. 2020. Artificial Intelligence: A Modern Approach (4th Edition). Pearson. http://aima.cs.berkeley.edu/.

Sutton, Richard S., and Andrew G. Barto. 2018. Reinforcement Learning: An Introduction. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.

See Also

Other solver: `solve_MDP_APPROX()`, `solve_MDP_DP()`, `solve_MDP_LP()`, `solve_MDP_SAMP()`, `solve_MDP_TD()`

Other MDP: `MDP()`, `Q_values()`, `absorbing_states()`, `act()`, `available_actions()`, `bellman_update()`, `convergence_horizon()`, `greedy_action()`, `gridworld`, `policy_evaluation()`, `regret()`, `sample_MDP()`, `start()`, `transition_graph()`, `transition_matrix()`, `unreachable_states()`, `value_function()`

Other MDPTF: `MDPTF()`, `absorbing_states()`, `act()`, `sample_MDP.MDPTF()`, `solve_MDP_APPROX()`, `start()`

Examples

```
data(Maze)
Maze

# default is value iteration (VI)
maze_solved <- solve_MDP(Maze)
maze_solved
policy(maze_solved)

# plot the value function U
plot_value_function(maze_solved)

# Gridworld solutions can be visualized
gw_plot(maze_solved)
```

---

solve_MDP_APPROX *Episodic Semi-gradient Sarsa with Linear Function Approximation*

Description

MDP control using state-value approximation. Semi-gradient Sarsa for episodic problems.

Usage

```
solve_MDP_APPROX(
  model,
  method = "semi_gradient_sarsa",
  horizon = NULL,
  discount = NULL,
  alpha = 0.01,
  epsilon = 0.2,
  n = 1000,
  w = NULL,
  ...,
  matrix = TRUE,
  continue = FALSE,
  progress = TRUE,
  verbose = FALSE
)

add_linear_approx_Q_function(model, ...)

## S3 method for class 'MDP'
add_linear_approx_Q_function(
  model,
  state_features = NULL,
  transformation = NULL,
```

```
  ...
)

## S3 method for class 'MDPTF'
add_linear_approx_Q_function(model, transformation = NULL, ...)

transformation_fourier(min, max, order, cs = expand.grid(0:order, 0:order))

approx_Q_value(model, state = NULL, action = NULL, w = NULL)

approx_greedy_action(model, state, w = NULL, epsilon = 0)

approx_greedy_policy(model, w = NULL)
```

Arguments

model	an MDP problem specification.
method	string; one of the following solution methods: 'semi_gradient_sarsa'
horizon	an integer with the number of epochs for problems with a finite planning horizon. If set to Inf, the algorithm continues running iterations till it converges to the infinite horizon solution. If NULL, then the horizon specified in model will be used.
discount	discount factor in range $(0, 1]$. If NULL, then the discount factor specified in model will be used.
alpha	step size.
epsilon	used for the ϵ-greedy behavior policies.
n	number of episodes used for learning.
w	a weight vector
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
progress	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.
state_features	a matrix with state features. Each row is the feature vector for a state.
transformation	a transformation function that is applied to the feature vector before it is used in the linear approximation.
min, max	vectors with the minimum and maximum values for each feature. This is used to scale the feature to the $[0, 1]$ interval for the Fourier basis.

order	order for the Fourier basis.
cs	an optional matrix or a data frame to specify cs values for the Fourier features selectively (overrides order).
state	a state (index or name)
action	an action (index or name)

Details

Linear Approximation:

The state-action value function is approximated by

$$\hat{q}(s, a) = \boldsymbol{w}^\top \phi(s, a),$$

where $\boldsymbol{w} \in \mathbb{R}^n$ is a weight vector and $\phi : S \times A \to \mathbb{R}^n$ is a feature function that maps each state-action pair to a feature vector. The gradient of the state-action function is

$$\nabla \hat{q}(s, a, \boldsymbol{w}) = \phi(s, a).$$

State-action Feature Vector Construction:

For a small number of actions, we can follow the construction described by Geramifard et al (2013) which uses a state feature function $\phi : S \to \mathbb{R}^m$ to construct the complete state-action feature vector. Here, we also add an intercept term. The state-action feature vector has length $1 + |A| \times m$. It has the intercept and then one component for each action. All these components are set to zero and only the active action component is set to $\phi(s)$, where s is the current state. For example, for the state feature vector $\phi(s) = (3, 4)$ and action $a = 2$ out of three possible actions $A = \{1, 2, 3\}$, the complete state-action feature vector is $\phi(s, a) = (1, 0, 0, 3, 4, 0, 0)$. The leading 1 is for the intercept and the zeros represent the two not chosen actions.

This construction is implemented in `add_linear_approx_Q_function()`.

Helper Functions:

The following helper functions for using approximation are available:

- `approx_Q_value()` calculates approximate Q values given the weights in the model or specified weights.
- `approx_greedy_action()` uses approximate Q values given the weights in the model or specified weights to find the the greedy action for a state.
- `approx_greedy_policy()` calculates the greedy-policy for the approximate Q values given the weights in the model or specified weights.

Episodic Semi-gradient Sarsa:

The implementation follows the algorithm given in Sutton and Barto (2018).

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

References

Sutton, Richard S., and Andrew G. Barto. 2018. Reinforcement Learning: An Introduction. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.

Alborz Geramifard, Thomas J. Walsh, Stefanie Tellex, Girish Chowdhary, Nicholas Roy, and Jonathan P. How. 2013. A Tutorial on Linear Function Approximators for Dynamic Programming and Reinforcement Learning. Foundations and Trends in Machine Learning 6(4), December 2013, pp. 375-451. doi:10.1561/2200000042

See Also

Other solver: solve_MDP(), solve_MDP_DP(), solve_MDP_LP(), solve_MDP_SAMP(), solve_MDP_TD()

Other MDPTF: MDPTF(), absorbing_states(), act(), sample_MDP.MDPTF(), solve_MDP(), start()

Examples

```
# Example 1: A maze without walls. The step cost is 1. The start is top-left and
# the goal (+100 reward) is bottom-right.
# This is the ideal problem for a linear approximation of the Q-function
# using the x/y location as state features.

m <- gw_maze_MDP(c(5, 5), start = "s(1,1)", goal = "s(5,5)")

# construct state features as the x/y coordinates in the gridworld
state_features <- gw_s2rc(S(m))
state_features

m <- add_linear_approx_Q_function(m, state_features)

# constructed state-action features (X) and approximate Q function
# and gradient
m$approx_Q_function

sol <- solve_MDP_APPROX(m, horizon = 1000, n = 10,
                    alpha = 0.01, epsilon = .5)
gw_plot(sol)
sol <- solve_MDP_APPROX(sol, horizon = 1000, n = 100,
          alpha = 0.01, epsilon = 0.05, verbose = FALSE, continue = TRUE)
gw_plot(sol)

### TESTS
sol <- solve_MDP_APPROX(m, horizon = 1000, n = 1,
                    alpha = 0.01, epsilon = .8, verbose = 2)
Q_values(sol)
gw_plot(sol)

gw_animate(m, method = "APPROX:semi", horizon = 1000, n = 10,
                    alpha = 0.01, epsilon = .8)

###

gw_matrix(sol, what = "value")
```

```
# learned weights and state values
sol$solution$w

# extracting approximate Q-values
approx_greedy_action(sol, "s(4,5)")
approx_Q_value(sol, "s(4,5)", "down")
approx_Q_value(sol)

# extracting a greedy policy using the approximate Q-values
approx_greedy_policy(sol)

# Example 2: Stuart Russell's 3x4 Maze using approximation
# The wall and the -1 absorbing state make linear approximation
# using just the position more difficult.
data(Maze)
gw_plot(Maze)

Maze_approx <- add_linear_approx_Q_function(Maze)
sol <- solve_MDP_APPROX(Maze_approx, horizon = 100, n = 100,
                     alpha = 0.01, epsilon = 0.3)
gw_plot(sol)

# Example 3: Use order-2 Fourier basis for features
order <- 2
cs <- expand.grid(0:order, 0:order)

# convert state features to Fourier basis features
x <- state2features(S(Maze))
x

x <- sweep(x, MARGIN = 2,
           STATS = apply(x, MARGIN = 2, max), FUN = "/")
x <- apply(cs, MARGIN = 1,
           FUN = function(c) cos(pi * x %*% c))
rownames(x) <- S(Maze)
x

Maze_approx <- add_linear_approx_Q_function(Maze, state_features = x)

sol <- solve_MDP_APPROX(Maze_approx, horizon = 100, n = 100, alpha = 0.1, epsilon = .5)
sol <- solve_MDP_APPROX(sol, horizon = 100, n = 1000, alpha = 0.1, epsilon = .1, continue = TRUE)
gw_plot(sol)

# order-2 fourier features can be specified easier
Maze_approx <- add_linear_approx_Q_function(Maze,
      transformation = transformation_fourier(min = c(0,0), max = c(3,4), order = 2))
sol <- solve_MDP_APPROX(Maze_approx, horizon = 1000, n = 200, alpha = 0.1, epsilon = .1)
gw_plot(sol)
```

`solve_MDP_DP` *Solve MDPs using Dynamic Programming*

Description

Solve MDPs via policy and value iteration.

Usage

```
solve_MDP_DP(
  model,
  method = "VI",
  horizon = NULL,
  discount = NULL,
  n = 1000L,
  error = 0.001,
  k_backups = 10L,
  V = NULL,
  ...,
  matrix = TRUE,
  continue = FALSE,
  verbose = FALSE,
  progress = TRUE
)
```

Arguments

`model`	an MDP problem specification.
`method`	string; one of the following solution methods: • `'VI'` - value iteration • `'PI'` - policy iteration • `'GenPS'`, `'PS_error'`, `'PS_random'` - prioritized sweeping
`horizon`	an integer with the number of epochs for problems with a finite planning horizon. If set to `Inf`, the algorithm continues running iterations till it converges to the infinite horizon solution. If `NULL`, then the horizon specified in `model` will be used.
`discount`	discount factor in range $(0, 1]$. If `NULL`, then the discount factor specified in `model` will be used.
`n`	maximum number of iterations allowed to converge. If the maximum is reached then the non-converged solution is returned with a warning.
`error`	value iteration: maximum Bellman error allowed for the convergence criterion.
`k_backups`	policy iteration: maximum number of Bellman backups used in the iterative policy evaluation step. Policy evaluation typically converges earlier with a maximum Bellman error less than `error`.

V	a vector with initial state values. If NULL, then the default of a vector of all 0s (V_zero()) is used.
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.
progress	logical; show a progress bar with estimated time for completion.

Details

The following dynamic programming methods are implemented using the algorithms presented in Russell and Norvig (2010).

- **(Modified) Policy Iteration** (Howard 1960; Puterman and Shin 1978) starts with a random policy and iteratively performs a sequence of
  1. (Approximate) policy evaluation to estimate the value function for the current policy. Iterative policy evaluation can be approximated by stopping early after k_backups iterations (see policy_evaluation(). In this case the algorithm is called *modified* policy iteration.
  2. Policy improvement is performed by updating the policy to be greedy (see greedy_policy()) with respect to the new value function. The algorithm stops when it converges to a stable policy (i.e., no changes between two iterations). Note that the policy typically stabilizes before the value function converges.
- **Value Iteration** (Bellman 1957) starts with an arbitrary value function (by default all 0s) and iteratively updates the value function for each state using the Bellman update equation (see bellman_update()).

$$v(s) \leftarrow \max_{a \in \mathcal{A}(s)} \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v(s')]$$

The iteration is terminated when the solution converges or the maximum of n iterations has been reached. Approximate convergence is achieved for discounted problems (with $\gamma < 1$) when the maximal value function change for any state δ is

$$\delta \leq \frac{error(1-\gamma)}{\gamma}.$$

It can be shown that this means that no state value is more than $error$ from the value in the optimal value function. For undiscounted problems, we use $\delta \leq error$.

A greedy policy is extracted from the final value function. Value iteration can be seen as policy iteration with policy evaluation truncated to one step.

- **Prioritized Sweeping** (Moore and Atkeson, 1993; Andre et al., 1997; Li and Littman, 2008) approximate the optimal value function by iteratively adjusting one state at a time. While value and policy iteration sweep in every iteration through all states, prioritized sweeping updates states in the order given by their priority. The priority reflects how much a state value may change given the most recently updated other states that can be directly reached via an action. This update order often lead to faster convergence compared to sweeping the whole state space in regular value iteration.

  We implement the two priority update strategies described as **PS** and **GenPS** by Li and Littman (2008).

  - **PS** (Moore and Atkeson, 1993) updates the priority of a state $H(s)$ using:

$$\forall s \in \mathcal{S} : H_{t+1}(s) \leftarrow \begin{cases} \max(H_t(s), \Delta_t \max_{a \in \mathcal{A}}(p(s_t|s,a)) \text{ for } s \neq s_{t+1} \\ \Delta_t \max_{a \in A}(p(s_t|s,a) \text{ for } s = s_{t+1} \end{cases}$$

    where $\Delta_t = |V_{t+1}(s_t) - V_t(s_t)| = |E(s_t; V_{t+1})|$, i.e., the Bellman error for the updated state.
  - **GenPS** (Andre et al., 1997) updates all state priorities using their current Bellman error:

$$\forall s \in \mathcal{S} : H_{t+1}(s) \leftarrow |E(s; V_{t+1})|$$

    where $E(s; V_{t+1}) = \max_{a \in A} \left[R(s,a) + \gamma \sum_{s \in S} p(s'|s,a) V(s')\right] - V(s)$ is a state's Bellman error.

  The update method can be chosen using the additional parameter `H_update` as the character string `"PS_random"`, `"PS_error"` or `"GenPS"`. The default is `H_update = "GenPS"`. For PS, random means that the priority vector is initialized with random values (larger than 0), and error means they are initialized with the Bellman error as in GenPS. However, this requires one complete sweep over all states.

  This implementation stops updating when the largest priority values over all states is less than the specified `error`.

  Since the algorithm does not sweep through the whole state space for each iteration, `n` is converted into an equivalent number of state updates $n = n\,|S|$.

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

References

Andre, D., Friedman, N., and Parr, R. 1997. "Generalized prioritized sweeping." In Advances in Neural Information Processing Systems 10, pp. 1001-1007. NeurIPS Proceedings

Bellman, Richard. 1957. "A Markovian Decision Process." Indiana University Mathematics Journal 6: 679-84. https://www.jstor.org/stable/24900506.

Howard, R. A. 1960. "Dynamic Programming and Markov Processes." Cambridge, MA: MIT Press.

Li, Lihong, and Michael Littman. 2008. "Prioritized Sweeping Converges to the Optimal Value Function." DCS-TR-631. Rutgers University. doi:10.7282/T3TX3JSX

Moore, Andrew, and C. G. Atkeson. 1993. "Prioritized Sweeping: Reinforcement Learning with Less Data and Less Real Time." Machine Learning 13 (1): 103–30. doi:10.1007/BF00993104.

Puterman, Martin L., and Moon Chirl Shin. 1978. "Modified Policy Iteration Algorithms for Discounted Markov Decision Problems." Management Science 24: 1127-37. doi:10.1287/mnsc.24.11.1127.

Russell, Stuart J., and Peter Norvig. 2020. Artificial Intelligence: A Modern Approach (4th Edition). Pearson. http://aima.cs.berkeley.edu/.

See Also

Other solver: solve_MDP(), solve_MDP_APPROX(), solve_MDP_LP(), solve_MDP_SAMP(), solve_MDP_TD()

Examples

```
data(Maze)

maze_solved <- solve_MDP(Maze, method = "DP:VI", verbose = TRUE)
policy(maze_solved)

# use prioritized sweeping (which is known to be fast for mazes)
maze_solved <- solve_MDP(Maze, method = "DP:GenPS", verbose = TRUE)
policy(maze_solved)

# finite horizon
maze_solved <- solve_MDP(Maze, method = "DP:VI", horizon = 3)
policy(maze_solved)
gw_plot(maze_solved, epoch = 1)
gw_plot(maze_solved, epoch = 2)
gw_plot(maze_solved, epoch = 3)
```

---

solve_MDP_LP *Solve MDPs using Linear Programming*

Description

Solve discounted, infinite horizon MDPs via linear programming.

Usage

```
solve_MDP_LP(
  model,
  method = "LP",
  horizon = NULL,
  discount = NULL,
  inf = 1000,
  lpSolve_args = list(),
  ...,
  matrix = NULL,
  continue = FALSE,
  verbose = FALSE,
  progress = NULL
)
```

Arguments

model	an MDP problem specification.
method	string; one of the following solution methods: 'LP'
horizon	Only infinite-horizon MDPs with horizon = Inf are supported.
discount	only undiscounted MDPs with discount = 1 are supported.
inf	value used for infinity when calling lpSolve::lp(). This should me much larger than the largest absolute reward in the model.
lpSolve_args	a list with additional arguments passed on to lpSolve::lp().
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.
progress	not supported by this solver.

Details

A linear programming formulation was developed by Manne (1960) and further described by Puterman (1996). For the optimal value function, the Bellman equation holds:

$$v^*(s) = \max_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} p(s, a, s')[r(s, a, s') + \gamma v^*(s')] \; \forall a \in \mathcal{A}, s \in \mathcal{S}$$

The maximization problem can reformulate as a minimization with a linear constraint for each state action pair. The optimal value function can be found by solving the following linear program:

$$\min \sum_{s \in S} v(s)$$

subject to

$$v(s) \geq \sum_{s' \in \mathcal{S}} p(s, a, s')[r(s, a, s') + \gamma v(s')], \ \forall a \in \mathcal{A}, s \in \mathcal{S}$$

Note:

- The discounting factor has to be strictly less than 1.
- The used solver does not support infinity and a sufficiently large value needs to be used instead (see parameter `inf`).
- Additional parameters to to `solve_MDP` are passed on to `lpSolve::lp()`.

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

References

Manne, Alan. 1960. "On the Job-Shop Scheduling Problem." Operations Research 8 (2): 219-23. doi:10.1287/opre.8.2.219.

Puterman, Martin L. 1996. Markov decision processes: discrete stochastic dynamic programming. John Wiley & Sons.

See Also

Other solver: `solve_MDP()`, `solve_MDP_APPROX()`, `solve_MDP_DP()`, `solve_MDP_SAMP()`, `solve_MDP_TD()`

Examples

```
data(Maze)

# we change the discount to 0.9 since LP is only implemented for discounted MDPs.
maze_solved <- solve_MDP(Maze, discount = 0.9, method = "LP:LP", verbose = TRUE)
maze_solved
policy(maze_solved)
```

`solve_MDP_MC` *Solve MDPs using Monte Carlo Control*

Description

Solve MDPs using Monte Carlo control.

Usage

```
solve_MDP_MC(
  model,
  method = "exploring_starts",
  horizon = NULL,
  discount = NULL,
  n = 100,
  Q = NULL,
  epsilon = NULL,
  alpha = NULL,
  first_visit = TRUE,
  ...,
  matrix = TRUE,
  continue = FALSE,
  progress = TRUE,
  verbose = FALSE
)
```

Arguments

`model`	an MDP problem specification.
`method`	string; one of the following solution methods: • `'exploring_starts'` - on-policy MC control with exploring starts. • `'on_policy'` - on-policy MC control with an ϵ-greedy policy. • `'off_policy"'` - off-policy MC control using an ϵ-greedy behavior policy.
`horizon`	an integer with the number of epochs for problems with a finite planning horizon. If set to `Inf`, the algorithm continues running iterations till it converges to the infinite horizon solution. If `NULL`, then the horizon specified in `model` will be used.
`discount`	discount factor in range $(0, 1]$. If `NULL`, then the discount factor specified in `model` will be used.
`n`	number of episodes used for learning.
`Q`	an initial state-action value matrix. By default an all 0 matrix is used.
`epsilon`	used for the ϵ-greedy behavior policies.
`alpha`	step size as a function of the time step `t` and the number of times the respective Q-value was updated `n` or a scalar. For expected Sarsa, alpha is often set to 1.

first_visit	if TRUE then only the first visit of a state/action pair in an episode is used to update Q, otherwise, every-visit update is used.
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
progress	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.

Details

The idea is to estimate the action value function for a policy as the average of sampled returns.

$$q_\pi(s, a) = \mathbb{E}_\pi[R_i | S_0 = s, A_0 = a] \approx \frac{1}{n} \sum_{i=1}^{n} R_i$$

Monte Carlo control simulates a whole episode using the current behavior policy and uses the sampled reward to update the Q values. For on-policy methods, the behavior policy is updated to be greedy (i.e., optimal) with respect to the new Q values. Then the next episode is simulated till the predefined number of episodes is completed.

Implemented are the following temporal difference control methods described in Sutton and Barto (2018).

- **Monte Carlo Control with exploring Starts** learns the optimal greedy policy. It uses the same greedy policy for behavior and target (on-policy learning). After each episode, the policy is updated to be greedy with respect to the current Q values. To make sure all states/action pairs are explored, it uses exploring starts meaning that new episodes are started at a randomly chosen state using a randomly chooses action.
- **On-policy Monte Carlo Control** learns an epsilon-greedy policy which it uses for behavior and as the target policy (on-policy learning). An epsilon-greedy policy is used to provide exploration. For calculating running averages, an update with $\alpha = 1/n$ is used by default. A different update factor can be set using the parameter alpha as either a fixed value or a function with the signature function(t, n) which returns the factor in the range $[0, 1]$.
- **Off-policy Monte Carlo Control** uses for behavior an arbitrary soft policy (a soft policy has in each state a probability greater than 0 for all possible actions). We use an epsilon-greedy policy and the method learns a greedy policy using importance sampling. Note: This method can only learn from the tail of the sampled runs where greedy actions are chosen. This means that it is very inefficient in learning the beginning portion of long episodes. This problem is especially problematic when larger values for ϵ are used.

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

References

Sutton, Richard S., and Andrew G. Barto. 2018. Reinforcement Learning: An Introduction. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.

---

solve_MDP_SAMP *Solve MDPs using Random-Sampling*

Description

Solve MDPs via random-sampling. Implemented is Sutton and Barto's one-step random-sampling one-step tabular Q-planning.

Usage

```
solve_MDP_SAMP(
  model,
  method = "q_planning",
  horizon = NULL,
  discount = NULL,
  alpha = function(t, n) min(10/n, 1),
  n = 1000,
  Q = NULL,
  ...,
  matrix = TRUE,
  continue = FALSE,
  progress = TRUE,
  verbose = FALSE
)
```

Arguments

`model`	an MDP problem specification.
`method`	string; one of the following solution methods: `'q_planning'`

horizon	an integer with the number of epochs for problems with a finite planning horizon. If set to Inf, the algorithm continues running iterations till it converges to the infinite horizon solution. If NULL, then the horizon specified in model will be used.
discount	discount factor in range $(0, 1]$. If NULL, then the discount factor specified in model will be used.
alpha	step size as a function of the time step t and the number of times the respective Q-value was updated n or a scalar. For expected Sarsa, alpha is often set to 1.
n	number of updates performed.
Q	an initial state-action value matrix. By default an all 0 matrix is used.
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
progress	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.

Details

Random-sample one-step tabular Q-planning is a simple, not very effective, planning method shown as an illustration in Chapter 8 of Sutton and Barto (2018). It randomly selects a state/action pair and samples the following state s' to perform a single a one-step update:

$$Q(s, a) \leftarrow Q(s, a) + \alpha * (R + \gamma \max_{a}^{'}(Q(s', a')) - Q(s, a))$$

Value

solve_MDP() returns an object of class MDP which is a list with the model specifications (model), the solution (solution). The solution is a list with the elements:

- policy a list representing the policy graph. The list only has one element for converged solutions.
- converged did the algorithm converge (NA) for finite-horizon problems.
- delta final δ (value iteration and infinite-horizon only)
- iterations number of iterations to convergence (infinite-horizon only)

References

Sutton, Richard S., and Andrew G. Barto. 2018. Reinforcement Learning: An Introduction. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.

See Also

Other solver: solve_MDP(), solve_MDP_APPROX(), solve_MDP_DP(), solve_MDP_LP(), solve_MDP_TD()

---

solve_MDP_TD *Solve MDPs using Temporal Differencing*

Description

Solve MDPs using 1-step and n-step tabular temporal difference control methods like q-learning and Sarsa.

Usage

```
solve_MDP_TD(
  model,
  method = "q_learning",
  horizon = NULL,
  discount = NULL,
  alpha = function(t, n) min(10/n, 1),
  epsilon = 0.2,
  n,
  Q = 0,
  ...,
  matrix = TRUE,
  continue = FALSE,
  progress = TRUE,
  verbose = FALSE
)

solve_MDP_TDN(
  model,
  method = "sarsa_on_policy",
  horizon = NULL,
  discount = NULL,
  n_step,
  alpha = function(t, n) min(10/n, 1),
  epsilon = 0.2,
  n,
  Q = 0,
  matrix = TRUE,
  continue = FALSE,
  progress = TRUE,
  verbose = FALSE,
  ...
)
```

Arguments

model	an MDP problem specification.
method	string; one of the following solution methods: • for TD: "sarsa", "q_learning", or "expected_sarsa" • for TDN: "sarsa_on_policy", or "sarsa_off_policy"
horizon	an integer with the number of epochs for problems with a finite planning horizon. If set to Inf, the algorithm continues running iterations till it converges to the infinite horizon solution. If NULL, then the horizon specified in model will be used.
discount	discount factor in range $(0, 1]$. If NULL, then the discount factor specified in model will be used.
alpha	step size as a function of the time step t and the number of times the respective Q-value was updated n or a scalar. For expected Sarsa, alpha is often set to 1.
epsilon	used for the ϵ-greedy behavior policies.
n	number of episodes used for learning.
Q	an initial state-action value matrix. By default an all 0 matrix is used.
...	further parameters are passed on to the solver function.
matrix	logical; if TRUE then matrices for the transition model and the reward function are taken from the model first. This can be slow if functions need to be converted or do not fit into memory if the models are large. If these components are already matrices, then this is very fast. For FALSE, the transition probabilities and the reward is extracted when needed. This is slower, but removes the time and memory requirements needed to calculate the matrices.
continue	logical; show a progress bar with estimated time for completion.
progress	logical; show a progress bar with estimated time for completion.
verbose	logical or a numeric verbose level; if set to TRUE or 1, the function displays the used algorithm parameters and progress information. Levels >1 provide more detailed solver output in the R console.
n_step	integer; steps for bootstrapping

Details

Implemented are several tabular temporal difference control methods described in Sutton and Barto (2018). Note that the MDP transition and reward models are used for these reinforcement learning methods only to sample from the environment.

The implementation uses an ϵ-greedy behavior policy, where the parameter epsilon controls the degree of exploration. The algorithms use a step size parameter α (learning rate). The learning rate alpha can be specified as a function with the signature function(t, n), where t is the number of episodes processed and n is the number of updates for the entry in the Q-table.

The general 1-step update is

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha[G_t - Q(S_t, A_t)],$$

where G_t is the target estimate for the given q-value. The different methods below estimate the target value differently.

If the model has absorbing states to terminate episodes, then no maximal episode length (`horizon`) needs to be specified. To make sure that the algorithm does finish in a reasonable amount of time, episodes are stopped after 1000 actions (with a warning). For models without absorbing states, the episode length has to be specified via `horizon`.

- **Q-Learning** (Watkins and Dayan 1992) is an off-policy temporal difference method that uses an ϵ-greedy behavior policy and learns a greedy target policy. The target value is estimated as the one-step bootstrapping using the target greedy policy:

$$G_t = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$$

- **Sarsa** (Rummery and Niranjan 1994) is an on-policy method that follows and learns the same policy. Here a an ϵ-greedy policy is used. The final ϵ-greedy policy is converted into a greedy policy. ϵ can be lowered over time (see parameter `continue`) to learn a greedy policy. The target is estimated as the one-step bootstrapping following the behavior policy:

$$G_t = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$$

- **Expected Sarsa** (Sutton and Barto 2018). We implement an on-policy Sarsa with an ϵ-greedy policy which uses the the expected value under the current policy for the update. It moves deterministically in the same direction as Sarsa would move in expectation.

$$G_t = R_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) Q(S_{t+1}, a)$$

  Because it uses the expectation, we can set the step size α to large values and 1 is common. The off-policy use of expected Sarsa simplifies to the Q-learning algorithm.

- **On and off-policy n-step Sarsa** (Sutton and Barto 2018). Estimate the return using the last n time steps:

$$G_{t:t+n} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n}, A_{t+n})$$

  $n = 1$ is regular 1-step Sarsa. Uaing $n = \inf$ is equivalent to Monte Carlo Control, howver, `solve_MDP_MC()` is more memory efficient.

  This estimate is used as the target for Sarsa. For the off-policy case, the update uses the importance sampling ratio. Note that updates are delayed n steps in this backward looking algorithm.

Value

`solve_MDP()` returns an object of class MDP which is a list with the model specifications (`model`), the solution (`solution`). The solution is a list with the elements:

- `policy` a list representing the policy graph. The list only has one element for converged solutions.
- `converged` did the algorithm converge (`NA`) for finite-horizon problems.
- `delta` final δ (value iteration and infinite-horizon only)
- `iterations` number of iterations to convergence (infinite-horizon only)

References

Rummery, G., and Mahesan Niranjan. 1994. "On-Line Q-Learning Using Connectionist Systems." Techreport CUED/F-INFENG/TR 166. Cambridge University Engineering Department.

Sutton, R. 1988. "Learning to Predict by the Method of Temporal Differences." Machine Learning 3: 9-44. https://link.springer.com/article/10.1007/BF00115009.

Sutton, Richard S., and Andrew G. Barto. 2018. Reinforcement Learning: An Introduction. Second. The MIT Press. http://incompleteideas.net/book/the-book-2nd.html.

Watkins, Christopher J. C. H., and Peter Dayan. 1992. "Q-Learning." Machine Learning 8 (3): 279-92. doi:10.1007/BF00992698.

See Also

Other solver: solve_MDP(), solve_MDP_APPROX(), solve_MDP_DP(), solve_MDP_LP(), solve_MDP_SAMP()

Examples

```
data(Maze)

# Example 1: Learn a Policy using Q-Learning
maze_learned <- solve_MDP(Maze, method = "TD:q_learning",
    epsilon = 0.2, n = 500, horizon = 100, verbose = TRUE)
maze_learned

policy(maze_learned)
plot_value_function(maze_learned)
gw_plot(maze_learned)

# Keep on learning, but with a reduced epsilon
maze_learned <- solve_MDP(maze_learned, method = "TD:q_learning",
    epsilon = 0.01, n = 500, horizon = 100, continue = TRUE, verbose = TRUE)

policy(maze_learned)
plot_value_function(maze_learned)
gw_plot(maze_learned)

# Example 2: n-step Sarsa
maze_learned <- solve_MDP(Maze, method = "TDN:sarsa_on_policy",
    n_step = 3, n = 10, horizon = 100, verbose = TRUE)
maze_learned

gw_plot(maze_learned)
```

---

start *Sample a Start State*

Description

Samples a start state which can be used for simulation.

Usage

```
start(model)
```

Arguments

model	an MDP model.

Value

a start state.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), transition_graph(), transition_matrix(), unreachable_states(), value_function()

Other MDPTF: MDPTF(), absorbing_states(), act(), sample_MDP.MDPTF(), solve_MDP(), solve_MDP_APPROX()

Examples

```
data(Maze)

start(Maze)
```

---

transition_graph *Transition Graph*

Description

Returns the transition model as an **igraph** object.

Usage

```
transition_graph(
  x,
  action = NULL,
  state_col = NULL,
  simplify_transitions = TRUE,
  remove_unavailable_actions = TRUE
)

plot_transition_graph(
  x,
```

```
  action = NULL,
  state_col = NULL,
  simplify_transitions = TRUE,
  main = NULL,
  ...
)

curve_multiple_directed(graph, start = 0.3)
```

Arguments

x	object of class MDP.
action	the name or id of an action or a set of actions. By default the transition model for all actions is returned.
state_col	colors used to represent the states.
simplify_transitions	logical; combine parallel transition arcs into a single arc.
remove_unavailable_actions	logical; don't show arrows for unavailable actions.
main	a main title for the plot.
...	further arguments are passed on to igraph::plot.igraph().
graph	The input graph.
start	The curvature at the two extreme edges.

Details

The transition model of an MDP is a Markov Chain. This function extracts the transition model as an igraph object.

Value

returns the transition model as an igraph object.

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_matrix(), unreachable_states(), value_function()

Examples

```
data("Maze")

g <- transition_graph(Maze)
g

plot_transition_graph(Maze)
plot_transition_graph(Maze,
```

```
  vertex.size = 20,
  edge.label.cex = .1, edge.arrow.size = .5, margin = .5
)

## Plot using the igraph library
library(igraph)
plot(g)

# plot with a different layout
plot(g,
  layout = igraph::layout_with_sugiyama,
  vertex.size = 20,
  edge.label.cex = .6
)

## Use visNetwork (if installed)
if (require(visNetwork)) {
  g_vn <- toVisNetworkData(g)
  nodes <- g_vn$nodes
  edges <- g_vn$edges

  visNetwork(nodes, edges) %>%
    visNodes(physics = FALSE) %>%
    visEdges(smooth = list(type = "curvedCW", roundness = .6), arrows = "to")
}
```

`transition_matrix` *Access to Parts of the Model Description*

Description

Functions to provide uniform access to different parts of the MDP problem description.

Usage

```
transition_matrix(
  model,
  action = NULL,
  start.state = NULL,
  end.state = NULL,
  ...,
  sparse = NULL,
  drop = TRUE,
  simplify = FALSE,
  trans_keyword = TRUE
)

reward_matrix(
  model,
```

```
  action = NULL,
  start.state = NULL,
  end.state = NULL,
  ...,
  sparse = NULL,
  drop = TRUE,
  simplify = FALSE
)

start_vector(model, start = NULL, sparse = NULL)

normalize_MDP(
  model,
  transition_prob = TRUE,
  reward = TRUE,
  start = FALSE,
  sparse = NULL,
  precompute_absorbing = TRUE,
  check_and_fix = FALSE,
  progress = TRUE
)
```

Arguments

model	A MDP object.
action	name or index of an action.
start.state, end.state	name or index of the state.
...	further arguments are passed on.
sparse	logical; use sparse matrix representation? NULL decides the representation based on the memory it would take to store the faster dense representation.
drop	logical; drop matrices to vectors when one row/one column is selected.
simplify	logical; try to simplify action lists into a vector or matrix?
trans_keyword	logical; translate keywords like "uniform" into matrices.
start	logical; convert the start probability distribution into a vector.
transition_prob	logical; convert the transition probabilities into a list of matrices.
reward	logical; convert the reward model into a list of matrices.
precompute_absorbing	logical; should absorbing states be precalculated?
check_and_fix	logical; checks the structure of the problem description.
progress	logical; show a progress bar with estimated time for completion.

Details

Several parts of the MDP description can be defined in different ways. In particular, the fields `transition_prob`, `reward`, and `start` can be defined using matrices, data frames, keywords, or functions. See MDP for details. The functions provided here, provide unified access to the data in these fields to make writing code easier.

Transition Probabilities $p(s'|s,a)$:

`transition_matrix()` accesses the transition model. The complete model is a list with one element for each action. Each element contains a states x states matrix with s (`start.state`) as rows and s' (`end.state`) as columns. Matrices with a low density can be requested in sparse format (as a Matrix::dgRMatrix). It is recommended to load package `MatrixExtra` to work with sparse matrices.

Reward $r(s,s',a)$:

`reward_matrix()` accesses the reward model. The preferred representation is a data.frame with the columns `action`, `start.state`, `end.state`, and `value`. This is a sparse representation.

The dense representation is a list of lists of matrices. The list levels are a (`action`) and s (`start.state`). The matrices are column vectors with rows representing s' (`end.state`).

To represent the rewards as a sparse matrices, **rewards that correspond to a transition with probability zero are zeroed out if the transition model** is stored as a list of matrices. This makes the reward matrices as sparse as the transition matrices. The function `normalize_MDP()` with `sparse = TRUE` will perform this representation.

Start state:

`start_vector()` translates the start state description into a probability vector.

Sparse Matrices and Normalizing MDPs:

Different components can be specified in various ways. It is often necessary to convert each component into a specific form (e.g., a dense matrix) to save time during access. Convert the Complete MDP Description into a consistent form `normalize_MDP()` converts all components of the MDP description into a consistent form and returns a new MDP definition where `transition_prob`, `reward`, and `start` are normalized. This includes the internal representation (dense, sparse, as a data.frame) and also, `states`, and `actions` are ordered as given in the problem definition to make safe access using numerical indices possible. Normalized MDP descriptions can be used in custom code that expects consistently a certain format.

The default behavior of `sparse = NULL` uses parse matrices for large models where the dense transition model would need more than `options("MDP_SPARSE_LIMIT")` (the default is about 100 MB which can be changed using `options()`). Smaller models use faster dense matrices.

Value

A list or a list of lists of matrices.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), unreachable_states(), value_function()

Examples

```
data("Maze")
gw_matrix(Maze)

# here is the internal structure of the Maze object
str(Maze)

# List of |A| transition matrices. One per action in the from start.states x end.states
Maze$transition_prob
transition_matrix(Maze)
transition_matrix(Maze, action = "up", sparse = FALSE)
transition_matrix(Maze,
  action = "up",
  start.state = "s(3,1)", end.state = "s(2,1)"
)

# List of list of reward matrices. 1st level is action and second level is the
#  start state in the form of a column vector with elements for end states.
Maze$reward
reward_matrix(Maze)
reward_matrix(Maze, sparse = TRUE)
reward_matrix(Maze,
  action = "up",
  start.state = "s(3,1)", end.state = "s(2,1)"
)

# Translate the initial start probability vector
Maze$start
start_vector(Maze, sparse = FALSE)
start_vector(Maze, sparse = "states")
start_vector(Maze, sparse = "index")

# Normalize the whole model using sparse representation
Maze_norm <- normalize_MDP(Maze, sparse = TRUE)
str(Maze_norm)

# Note to make the reward matrix sparse, all rewards
# for transitions with probability of 0 are zeroed out.
reward_matrix(Maze_norm)
```

---

unreachable_states *Unreachable States*

Description

Find or removes unreachable states using the transition model.

Usage

```
unreachable_states(
  model,
  horizon = Inf,
  sparse = "states",
  progress = TRUE,
  ...
)

remove_unreachable_states(model, ...)
```

Arguments

model	a MDP object.
horizon	only states that can be reached within the horizon are reachable.
sparse	logical; return a sparse logical vector?
progress	logical; show a progress bar?
...	further arguments are passed on.

Details

The function unreachable_states() checks if states cannot be reached from any other state. It performs a depth-first search which can be slow. The search breaks cycles to avoid an infinite loop. The search depth can be restricted using horizon.

The function remove_unreachable_states() simplifies a model by removing unreachable states from the model description.

Value

unreachable_states() returns a logical vector indicating the unreachable states.

remove_unreachable_states() returns a model with all unreachable states removed.

Author(s)

Michael Hahsler

See Also

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), value_function()

Examples

```
# create a Maze with an unreachable state

maze_unreach <- gw_read_maze(
    textConnection(c("XXXXXX",
                     "XS X X",
                     "X  XXX",
                     "X   GX",
                     "XXXXXX")))
gw_plot(maze_unreach)

unreachable_states(maze_unreach)
unreachable_states(maze_unreach, sparse = FALSE)

maze <- remove_unreachable_states(maze_unreach)
unreachable_states(maze)
gw_plot(maze)
```

`value_function` *Value Function*

Description

Extracts the value function from a solved MDP.

Usage

```
value_function(model, drop = TRUE)

plot_value_function(
  model,
  epoch = 1L,
  legend = TRUE,
  col = NULL,
  ylab = "Value",
  las = 3,
  main = NULL,
  ...
)

V_zero(model, value = 0)

V_random(model, min = 1e-06, max = 1)
```

Arguments

`model`	a solved MDP.
`drop`	logical; drop the list for converged, epoch-independent value functions.

epoch	epoch for finite time horizon solutions.
legend	logical; show legend.
col, ylab, las	are passed on to graphics::barplot().
main	a main title for the plot. Defaults to the name of the problem.
...	further arguments are passed on to graphics::barplot()'.
value	value to initialize the value function with. Default is 0.
min, max	minimum and maximum for the uniformly distributed random state values.

Value

Returns the value function as a numeric vector with one value for each state or as a matrix with rows for states and columns for epochs.

Author(s)

Michael Hahsler

See Also

Other policy: Q_values(), action(), bellman_update(), greedy_action(), policy(), policy_evaluation(), regret(), reward(), visit_probability()

Other MDP: MDP(), Q_values(), absorbing_states(), act(), available_actions(), bellman_update(), convergence_horizon(), greedy_action(), gridworld, policy_evaluation(), regret(), sample_MDP(), solve_MDP(), start(), transition_graph(), transition_matrix(), unreachable_states()

Examples

```
data("Maze")
sol <- solve_MDP(Maze)
sol

value_function(sol)
plot_value_function(sol)

## finite-horizon problem
sol <- solve_MDP(Maze, horizon = 3)
policy(sol)
value_function(sol)
plot_value_function(sol, epoch = 1)
plot_value_function(sol, epoch = 2)
plot_value_function(sol, epoch = 3)

# For a gridworld we can also plot is like this
gw_plot(sol, epoch = 1)
gw_plot(sol, epoch = 2)
gw_plot(sol, epoch = 3)
```

visit_probability *State Visit Probability*

Description

Calculates the state visit probability (the modified stationary distribution) when following a policy from the start state.

Usage

```
visit_probability(model, pi = NULL, start = NULL, method = "power", ...)
```

Arguments

model	a solved MDP object.
pi	the used policy. If missing the policy in model is used.
start	specification of the start distribution. If missing the specification in model is used.
method	calculate the modified stationary distribution using "power" (power iteration) or "sample" (trajectory sampling).
...	further arguments are passed on to the internal implementations (see Details section).

Details

The visit probability is the stationary distribution for the transition matrix induced by the policy. To account for absorbing states, we modify the transition matrix by setting all outgoing probabilities from absorbing states to 0.

Power iteration:

The stationary distribution can be estimated as the sum of multiplying the start distribution repeatedly with the modified transition matrix induced by the policy. We stop multiplying when the largest difference between entries in the two consecutive vectors is less then the extra parameter:

- min_err stop criterion for the power iteration (default: 1e-6).
- sparse logical; should a sparse transition matrix be used for the power iteration?

The resulting vector is normalized to probabilities.

Sample method:

The stationary distribution is calculated using n random walks. The state visit counts are normalized to a probabilities. Additional parameters are:

- n number of random walks (default 1000).
- horizon maximal horizon used to stop a random walk if it has not reached an absorbing state.

Value

a visit probability vector over all states.

Author(s)

Michael Hahsler

See Also

Other policy: `Q_values()`, `action()`, `bellman_update()`, `greedy_action()`, `policy()`, `policy_evaluation()`, `regret()`, `reward()`, `value_function()`

Examples

```
data("Maze")
Maze

sol <- solve_MDP(Maze)
visit_probability(sol)

# gw_matrix also can calculate the visit_probability.
gw_matrix(sol, what = "visit_probability")
```

Windy_gridworld *Windy Gridworld MDP Windy Gridworld MDP*

Description

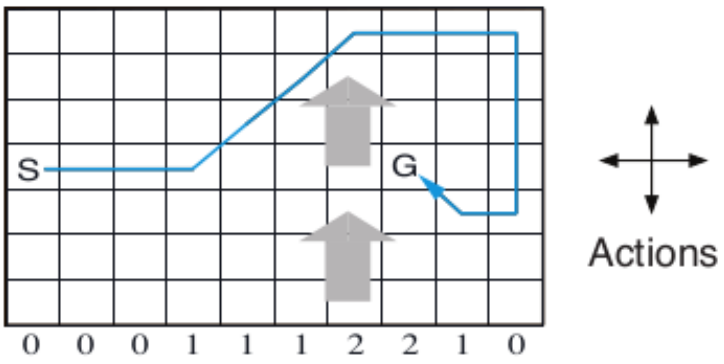
The Windy gridworld MDP example from Chapter 6 of the textbook "Reinforcement Learning: An Introduction."

Format

An object of class MDP.

Details

The gridworld has the following layout:



The grid world is represented as a 7 x 10 matrix of states. In the middle region the next states are shifted upward by wind (the strength in number of squares is given below each column). For example, if the agent is one cell to the right of the goal, then the action left takes the agent to the cell just above the goal.

No discounting is used (i.e., $\gamma = 1$).

References

Richard S. Sutton and Andrew G. Barto (2018). Reinforcement Learning: An Introduction Second Edition, MIT Press, Cambridge, MA.

See Also

Other MDP_examples: Cliff_walking, DynaMaze, MDP(), Maze

Other gridworld: Cliff_walking, DynaMaze, Maze, gridworld

Examples

```
data(Windy_gridworld)
Windy_gridworld

gw_matrix(Windy_gridworld)
gw_matrix(Windy_gridworld, what = "labels")

gw_plot(Windy_gridworld)

# The Goal is an absorbing state
absorbing_states(Windy_gridworld, sparse = "states")

# visualize the transition graph
gw_plot_transition_graph(Windy_gridworld)

# solve using value iteration
sol <- solve_MDP(Windy_gridworld)
sol
policy(sol)
gw_plot(sol)
```

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